Ma1023 Test 1

Calculus III

A Term, 2013

Name: ____________________

1. (3 pts) Suppose $f(x)$ is a function for all x and that

$$\int_0^\infty f(x)\,dx = \infty.$$

Label each of the following with **T** if it must be true, **F** if it must be false, and **X** if it cannot be determined from the given information.

____ $\displaystyle\int_0^\infty 1 - 3f(x)\,dx = -\infty$

X - Linearity of the integral. The integral of 1 is ∞, so altogether we have the indeterminant form $\infty - \infty$

____ $\displaystyle\int_0^\infty f(x) + f(-x)\,dx$ does not exist

X - substitution gives $\int_0^L f(-x)\,dx = -\int_0^{-L} f(x)\,dx$ and we have no information about $f(x)$ on the negative x axis.

____ $\displaystyle\int_0^\infty f(x)^2\,dx = \infty$

X - there is no product rule for integrals. There is no way to tell.

2. (3 pts) Compute carefully and neatly the following integral, showing all steps required.

$$\int_3^\infty \frac{x^2}{\sqrt{1+5x^3}}\,dx = \lim_{L\to\infty} (2/15)\sqrt{1+5L^3} - (2/15)\sqrt{1+15^3} = \infty$$

Antiderivative: $\displaystyle\int \frac{x^2}{\sqrt{1+5x^3}}\,dx \overset{\substack{u=1+5x^3\\ du=15x^2dx}}{=} (1/15)\int u^{-1/2}\,du$

$$= (1/15)\left(\frac{u^{1/2}}{1/2}\right) = (2/15)\sqrt{u} = (2/15)\sqrt{1+5x^3}$$

3. (4 pts) Compute carefully and neatly the following integral, showing all steps required.

$$\int_2^\infty xe^{-5x}\,dx = \lim_{L\to\infty}[-Le^{-5L}/5 - e^{-5L}/25] - [-2e^{-10}/5 - e^{-10}/25]$$

$$= [(1/5)\lim_{L\to\infty}\frac{L}{e^{5L}}] - 0 + [2e^{-10}/5 + e^{-10}/25]$$

$$\overset{\text{by l'H}}{=} [(1/5)\lim_{L\to\infty}\frac{1}{5e^{5L}}] - 0 + [2e^{-10}/5 + e^{-10}/25]$$

$$= 0 - 0 + [2e^{-10}/5 + e^{-10}/25] = 2e^{-10}/5 + e^{-10}/25 = 11e^{-10}/25$$

Antiderivatives: $\displaystyle\int xe^{-5x}\,dx \overset{\substack{u=x \quad dv=e^{-5x}dx\\ du=dx \quad v=e^{-5x}/-5}}{=} -xe^{-5x}/5 - \int (e^{-5x}/-5)\cdot 1\,dx$

$$= -xe^{-5x}/5 + (1/5)\int e^{-5x}\,dx = -xe^{-5x}/5 + (1/5)e^{-5x}/-5$$

$$= -xe^{-5x}/5 - e^{-5x}/25$$

$$\int e^{-5x}\,dx \overset{\substack{u=-5x\\ du=-5dx}}{=} (-1/5)\int e^u\,du = (-1/5)e^u = (-1/5)e^{-5x}$$