Ma1023

Test 1

Calculus III

A Term, 2013

Name: ____________________

1. (4 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to 0} \frac{1-\cos(3x)}{\sin^2(x)}$$

is of the form $\frac{0}{0}$. L'Hopital's rule says to consider

$\lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos(3x))}{\frac{d}{dx}\sin^2(x)} = \lim_{x\to 0} \frac{3\sin(3x)}{2\sin(x)\cos(x)}$ which is also of the form $\frac{0}{0}$. So L'Hopital's rule says to consider

$$\lim_{x\to 0} \frac{\frac{d}{dx}3\sin(3x)}{\frac{d}{dx}2\sin(x)\cos(x)} = \lim_{x\to 0} \frac{9\cos(3x)}{2(\cos^2(x)-\sin^2(x))} = \frac{9\cdot 1}{2(1+0)} = \frac{9}{2}$$

So the original limit is $\frac{1}{2}$.

[This is a rather formal write-up such as you see in your text.]

2. (3 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to\infty} \frac{x^4}{(1-x^2)(1+x^2)} = \lim_{x\to\infty} \frac{x^4}{1-x^4} \overset{\text{by l'H}}{=} \lim_{x\to\infty} \frac{4x^3}{-4x^4} = \lim_{x\to\infty} -1 = -1$$

[This is the calculation style I showed you in class.]

3. (3 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to 0^+} (2x)^{3x}$$

$$= \lim_{x\to 0^+} e^{3x\ln(2x)} = \lim_{x\to 0^+} e^{3\frac{\ln(2x)}{1/x}}$$

$$\overset{\text{by l'H}}{=} \lim_{x\to 0^+} e^{3\frac{2/2x}{-1/x^2}} = \lim_{x\to 0^+} e^{-3x} = e^0 = 1$$