Ma1023 **Calculus III** A Term, 2013

Test 1 Name: ____________________

1. (4 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$\lim_{x\to 0} \frac{1-\cos(x)}{\sin(x^2)}$ is of the form $\frac{0}{0}$. L'Hopital's rule says to consider

$\lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos(x))}{\frac{d}{dx}\sin(x^2)} = \lim_{x\to 0} \frac{\sin(x)}{2x\cos(x^2)}$ which is also of the form $\frac{0}{0}$. So L'Hopital's rule says to consider

$$\lim_{x\to 0} \frac{\frac{d}{dx}\sin(x)}{\frac{d}{dx}2x\cos(x^2)} = \lim_{x\to 0} \frac{\cos(x)}{2(x\sin(x^2)(2x)+\cos(x^2))} = \frac{1}{2(0+1)} = \frac{1}{2}$$

So the original limit is $\frac{1}{2}$.

[This is a rather formal write-up such as you see in your text.]

2. (3 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to\infty} \frac{x^3}{1-x-x^3} \overset{\text{by l'H}}{=} \lim_{x\to\infty} \frac{3x^2}{-1-3x^2} \overset{\text{by l'H}}{=} \lim_{x\to\infty} \frac{6x}{-6x} = -1$$

[This is the calculation style I showed you in class.]

3. (3 pts) Use l'Hopital's rule to compute the following limit. Show all work neatly and clearly.

$$\lim_{x\to 0^+} (4x)^{3x} = \lim_{x\to 0^+} e^{3x\ln(4x)} = \lim_{x\to 0^+} e^{3\frac{\ln(4x)}{1/x}}$$

$$\overset{\text{by l'H}}{=} \lim_{x\to 0^+} e^{3\frac{4/4x}{-1/x^2}} = \lim_{x\to 0^+} e^{-3x} = e^0 = 1$$