Ma1023 | Pseudo-Final Exam

Calculus III

A Term, 2013

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1. Mark the following **T** and **F** for false, and **X** if it cannot be determined from the given information.

____ $1^\infty = \frac{0}{0} = 1$.

____ If $\dfrac{d}{d\theta} r(\theta_0) = 0$ then the graph of $r(\theta)$ has a horizontal tangent at θ_0 in the xy-plane.

____ $\cos\left(1 + \dfrac{1}{\infty}\right) = 1$.

____ $r(\theta) = \sin(\theta)\cos(\theta)\tan(\theta)$ has as horizontal tangent at the origin in the xy plane.

____ If $\{a_n\}_{n=1}^\infty$ converges to π then $\sum_{n=1}^\infty a_n$ cannot converge to π.

____ The series $\sum_{n=1}^\infty 17$ is a geometric series.

____ Every telescoping series converges.

____ The limit comparison test fails if the limit is 1.

____ If $a_n > 0$ and $\lim\limits_{n\to\infty} a_n$ then $\sum_{n=1}^\infty (-1)^n a_n$ converges by the Alternating Series Test.

____ The power series $\sum_{n=1}^\infty (-1)^n \frac{n^{(x-2)}}{n!}$ has radius of convergence 1.

____ Every Taylor Series converges at its center.

____ The 5'th Maclauren Polynomial of $f(x) = |x|$ is actually a polynomial of degree 4.

____ If $\lim_{n\to\infty} R_n(x) \leq 1$, then the Taylor series converges.

____ If vector is directed into the 4'th quadrant then its magnitude is negative,

____ For all vectors $\mathbf{A}\cdot\mathbf{B} = -\mathbf{B}\cdot\mathbf{A}$.

____ The plane $(1, 2, 1)\cdot\mathbf{X} = -1$ has distance 1 from the origin.

2. Determine for which values of x the following series converges absolutely, converges conditionally, or diverges. Show all steps required to carefully present your work.

$$\sum_{n=0}^{\infty} \frac{n^3}{1+n^4}(x+1)^n$$

Also compute the radius of convergence.

3. Determine for which values of x the following series converges absolutely, converges conditionally, or diverges. Show all steps required to carefully present your work.

$$\sum_{n=0}^{\infty} \frac{x^{3n}}{n!}$$

Also compute the radius of convergence.

In the interval of converge define $f(x) = \sum_{n=0}^{\infty} \frac{x^{3n}}{2^n n!}$

For the above series, answer the following:

a) Find $f'''(0)$.

b) Compute $f'(x)$ inside the radius of convergence.

c) Compute $\int f(x)\, dx$ inside the radius of convergence.

4.

a) Find the limit of the sequence $\left\{\frac{n^2}{2^n}\right\}$.

Show all steps required to carefully present your work.

b) Decide whether or not the series $\sum_{n=0}^{\infty} \frac{n^2}{2^n}$ is geometric. If it is geometric, and converges, determine the sum.

Show all steps required to carefully present your work.

c) Using the Taylor Series for $\sin(x)$ with center 0, find the Taylor series for $\sin(3x^2)$.

Show all steps required to carefully present your work.

5. Let $\mathbf{u} = 3\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{v} = 1\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}$.

a) Find the magnitudes of $\mathbf{u}$ and $\mathbf{v}$ as well as unit vectors which point in their directions.

b) Find the angle θ between $\mathbf{u}$ and $\mathbf{v}$.

c) Find $(\mathbf{u} + 2\mathbf{v}) \cdot (2\mathbf{u} + \mathbf{v})$.

d) Which of the following products is undefined, and why?

$(2\mathbf{u}) \cdot \mathbf{u}, \quad (2\mathbf{u}) \times \mathbf{u}, \quad (\mathbf{u} \cdot \mathbf{v}) \cdot \mathbf{v}, \quad (\mathbf{u} \cdot \mathbf{v}) \times \mathbf{v}, \quad (\mathbf{u} \times \mathbf{v}) \times \mathbf{v}, \quad (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{v}.$

d) Find the equation of the plane which is normal to $\mathbf{u}$ and whose closest distance to the origin is $|\mathbf{v}|$.

6. Let $\mathbf{f}(t) = t\mathbf{i} + t^2 - t^2\mathbf{j}$ be a position vector at time t.
a) Find all values t for which the speed is 0.

b) Show that the velocity vector at $t = 0$ is a unit vector.

c) Find a unit vector tangent to the curve at $t = 1$.

d) Find any vector perpendicular to both $\mathbf{f}(1)$ and $\mathbf{v}(1)$.