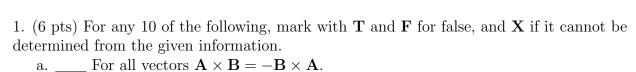
Ma1023 Final Exam (A)

## Calculus III

A Term, 2013

Print Name:



True. The cross product is anti-commutative.

b. \_\_\_\_ If 
$$\lim_{n\to\infty} \frac{f(n+1)}{f(n)} = 0$$
 then the series  $\sum_{n=0}^{\infty} f(n)$  converges.

True. Ratio Test.

c. \_\_\_\_ If the power series  $\sum_{n=0}^{\infty} c_k (x-8)^k$  converges absolutely at x=7, then it converges absolutely for x=5.

You cannot conclude it. The radius of convergence could be 2.

d. \_\_\_\_ For all vectors  $\mathbf{u}$ , it is true that  $\mathbf{u} \times \mathbf{0} = \mathbf{u}$ .

False. It is only true for  $\mathbf{u} = \mathbf{0}$ .

e. \_\_\_\_ The Taylor series for  $f(x) = \cos(x)$  has radius of convergence 1.

False. It is infinite.

f. \_\_\_\_ The plane  $(2,1,2) \cdot \mathbf{v} = 3$  has distance 1 from the origin.

True. (2/3, 1/3, 2/3) is a unit vector.

2. (8 pts) For each value of x determine whether the following power series converges absolutely, converges conditionally, or diverges. Show all steps required to carefully present your work.

It would.

$$\sum_{n=1}^{\infty} \frac{1}{n2^{n}} (x-1)^{n}$$

$$\sum_{k=1}^{\infty} \frac{1}{n2^{n}} (x-1)^{n}$$
Absolute Value Series  $\sum_{n=1}^{\infty} \frac{1}{n2^{n}}$ 

Ratio Test:  $|T_{nn}(\frac{1\times-1)^{n+1}}{(n+1)(2^{n+1})}| = |T_{nn}(\frac{1\times-1)}{n2^{n}}| = |T_{nn}(\frac{1\times-1)^{n}}{n2^{n}}| = |T_$ 

3. (4 pts) The power series  $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$  converges absolutely for all x, so define

$$f(x) = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$$

Answer each of the following:

- a) Find  $f^{(7)}(0)$ .
- b) Find the Taylor Series for f'(x) centered at 0. What is its radius of convergence?

a) Taylor Serice 
$$\sum_{k=0}^{\infty} f^{(k)}(a)(x-a)^k$$
 $k=0$ 
 $k=0$ 
 $k!$ 

The term  $f^{(q)}(o) x^{\frac{1}{2}}$  for  $\sum_{n=0}^{\infty} x^{2n+1}$  the term in  $x^{\frac{1}{2}} + x^{\frac{1}{2}}$ 
 $f^{(q)}(a) x^{\frac{1}{2}} = x^{\frac{1}$ 

4. (4 pts) Using the known Taylor Series for  $e^x$  with center 0, find the Taylor series for the function  $f(x) = x^2 e^{3x+1}$  centered at 0.

Show all steps required to carefully present your work.

Recall 
$$e^{x} = \sum_{k=0}^{\infty} \frac{x^{n}}{n!}$$

$$x^{2}e^{3x+1} = x^{2}e^{3x}e = x \sum_{k=0}^{\infty} \frac{3x^{n}}{n!}e = \sum_{k=0}^{\infty} \frac{e^{3^{n}}x^{n}x^{2}}{n!}e = \sum_{k=0}^{\infty} \frac{e^{3^{n}}x^{n}x^{2}}{n!}e$$

- 5. (8 pts) Let  $\mathbf{f}(t) = \cos(\pi t^2)\mathbf{i} + \sin(\pi t^2)\mathbf{j} + t^2\mathbf{k}$  be a position vector at time t.
- a) Find all times where the speed is 0.
- b) Show that the angle between the velocity vector  $\mathbf{v}(t)$  and the vector  $\mathbf{k}$  is constant for all t > 0.
  - c) Find a unit vector tangent to the curve at t = 2.
  - d) Compute  $|\mathbf{f}(2) \times \mathbf{v}(2)|$ .

a) 
$$\int_{Y(1)} f(1) = \cos(\pi t^{2}) \vec{l} + \sin(\pi t^{2}) \vec{l} + f^{2} \vec{k}$$

$$Velocity: Y(1) = -2\pi t \sin(\pi t^{2}) \vec{l} + 2\pi t \cos(\pi t^{2}) \vec{j} + 2t \vec{k}$$

$$\int_{Y(1)} f(1) = -2\pi t \sin(\pi t^{2}) \vec{l} + 2\pi t \cos(\pi t^{2}) \vec{j} + 2t \vec{k}$$

$$\int_{Y(1)} f(2) = -2\pi t \sin(\pi t^{2}) \vec{l} + 2\pi t \cos^{2}(\pi t^{2}) + 4t^{2}$$

$$= 2t t \sqrt{\pi^{2}} (\sin^{2}(\pi t^{2}) + \cos^{2}(\pi t^{2}) + 4t^{2}$$

$$= 2t t \sqrt{\pi^{2}} (\sin^{2}(\pi t^{2}) + \cos^{2}(\pi t^{2})) + \frac{1}{4}$$

$$= 2t t \sqrt{\pi^{2}} + 1$$

$$= 2t t \sqrt{\pi^{2}} + 2$$

$$= 2t t \sqrt$$

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