Ma1023 Calculus III A Term, 2010

Quiz 4

Print Name: ____________________

Sign: ____________________

1. (4 pts) Compute the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{(n+1)2^{2n}}$$

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{\frac{(x-3)^{n+1}}{(n+2)2^{2n+2}}}{\frac{(x-3)^n}{(n+1)2^{2n}}} \right| = \lim_{n\to\infty} \frac{|x-3|(n+2)}{2^2(n+1)} = \frac{|x-3|}{4}$$

So the power series converges absolutely when $|x-3|/4 < 1$, that is when $|x-3| < 4$ and diverges for $|x-3| > 4$, that is $|x-3| > 4$.

So the series converges for x within 4 units of the center, 3, and the Radius of Convergence is 4.

2. (3 pts) For the series in problem 1, find the interval of convergence. (Including the endpoints, if any.)

The only values of x not covered in Problem 1 are when $x-3=4$ and $x-3=-4$, that is when $x=7$ or $x=-1$.

For $x=7$, the series is $\displaystyle\sum_{n=0}^{\infty} \frac{(7-3)^n}{(n+1)2^{2n}} = \sum_{n=0}^{\infty} \frac{4^n}{(n+1)4^n} = \sum_{n=0}^{\infty} \frac{1}{n+1}$ which is a series of positive terms which we compare with the p-series $\sum 1/n$ using the Limit Comparison Test:

$$\lim_{n\to\infty} \frac{1/n}{1/(n+1)} = \lim_{n\to\infty} \frac{n+1}{n} = 1$$

which is neither 0 nor ∞, so the series $\sum_{n=0}^{\infty} 1/n+1$ converges by the Limit Comparison Test.

For $x=-1$, the series is $\displaystyle\sum_{n=0}^{\infty} \frac{(-1-3)^n}{(n+1)2^{2n}} = \sum_{n=0}^{\infty} \frac{(-4)^n}{(n+1)4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n 4^n}{(n+1)4^n} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ which is an alternating series, whose absolute value series we have just seen diverges. So it does not converge absolutely, and we check convergence with the Alternating Series Test. The factor $(-1)^n$ just alternates the signs.

The terms $1/(n+1)$ are decreasing since the reciprocals $n+1$ is increasing.

Also, $\displaystyle\lim_{n\to\infty} \frac{1}{n+1} = 0$ so the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n+1}$ converges by the Alternating series test, and the interval of convergence is

$$-1 \leq x < 7.$$

3. (2 pts) For the series in problems 1 and 2, set $f(x) = \sum_{n=0}^{\infty} \frac{(x-3)^n}{(n+1)2^{2n}}$ for all x in the interval of convergence. Compute $f'(3)$.

First we compute

$$f'(x) = \sum_{n=1}^{\infty} \frac{n(x-3)^{n-1}}{(n+1)2^{2n}}$$

where the sum starts from $n = 1$ since the constant term, exponent of $x - 3$ is zero, has derivative zero.

The new constant term is when $n = 1$, so the value of $f'(x)$ at the center $x = 0$ is

$$\frac{1}{(1+1)2^{2\cdot 1}} = \frac{1}{(2)2^2} = \frac{1}{8}$$

4. (1 pts) For the series in problems 1, 2 and 3, Express $f'(4)$ as an infinite series.

Using the derivative above

$$f'(4) = \sum_{n=1}^{\infty} \frac{n(4-3)^{n-1}}{(n+1)2^{2n}} = \sum_{n=1}^{\infty} \frac{n(1)^{n-1}}{(n+1)2^{2n}} = \sum_{n=1}^{\infty} \frac{n}{(n+1)2^{2n}}$$