Ma1023 Quiz 4

Calculus III

A Term, 2010

Print Name: ____________________

Sign: ____________________

1. (4 pts) Compute the $P_3(x)$, the 3rd Taylor polynomial of $f(x) = \ln(x)$ wit center $a = 1$.

$f(x) = \ln x \quad f(1) = 0$

$f'(x) = \frac{1}{x} = x^{-1} \quad f'(1) = 1$

$f''(x) = -1x^{-2} \quad f''(1) = -1$

$f'''(x) = 2x^{-3} \quad f'''(1) = 2$

$P_3(x) = 0 + \frac{1}{1!}(x-1) - \frac{1}{2!}(x-1)^2 + \frac{2}{3!}(x-1)^3$

$P_3(x) = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

Thanks to Keegan Westwater

2. (3 pts) Use $P_3(x)$ in problem 1 to approximate $\ln(1.1)$.

$P_3(1.1) = (1.1-1) - \frac{1}{2}(1.1-1)^2 + \frac{1}{3}(1.1-1)^3$

$P_3(1.1) = 0.1 - \frac{1}{2}(0.1)^2 + \frac{1}{3}(0.1)^3$

$\ln(1.1) \approx P_3(1.1) = 0.1 - \frac{1}{2}(0.01) + \frac{1}{3}(0.001)$

Thanks to Ali La Rue

3. (2 pts) Give the Remainder for the approximation if problem 2, (including the interval for 'z').

$R_3(x) = \frac{(x-1)^4}{4!} \cdot (-6z^{-4})$

$R_3(x) = -\frac{(x-1)^4}{4} \cdot z^{-4}$

$R_3(1.1) = -\frac{(1.1-1)^4}{4} \cdot z^{-4}$

$R_3(1.1) = -\frac{.0001}{4} z^{-4}$

Thanks to Matt Steeves

4. (1 pts) Show that the approximation in Problem 2 has 4 decimal place accuracy, that is, that the error is at most $.5 \times 10^{-4}$.

error $= |R_3(1.1)| = \frac{0.0001}{4z^4} \le \frac{0.0001}{4(1)^4} = 0.000025 = .25 \times 10^{-4}$

$|R_3(1.1)| \le .25 \times 10^{-4} < .5 \times 10^{-4}$ $\therefore$ the approx. in problem 2 has 4 decimal place accuracy

Thanks to Stephen Peters