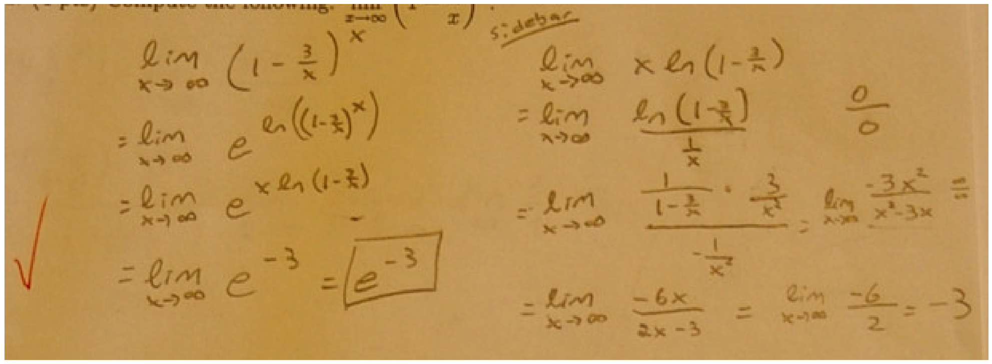
Ma1023 **Calculus III** A Term, 2010

Quiz 2 Name: ____________________

1. (4 pts) Compute the following: $\displaystyle\lim_{x\to\infty}\left(1-\frac{3}{x}\right)^x$.



$$\lim_{x\to\infty}\left(1-\tfrac{3}{x}\right)^x = \lim_{x\to\infty} e^{\ln\left(\left(1-\frac{3}{x}\right)^x\right)} = \lim_{x\to\infty} e^{x\ln\left(1-\frac{3}{x}\right)} = \lim_{x\to\infty} e^{-3} = \boxed{e^{-3}}$$

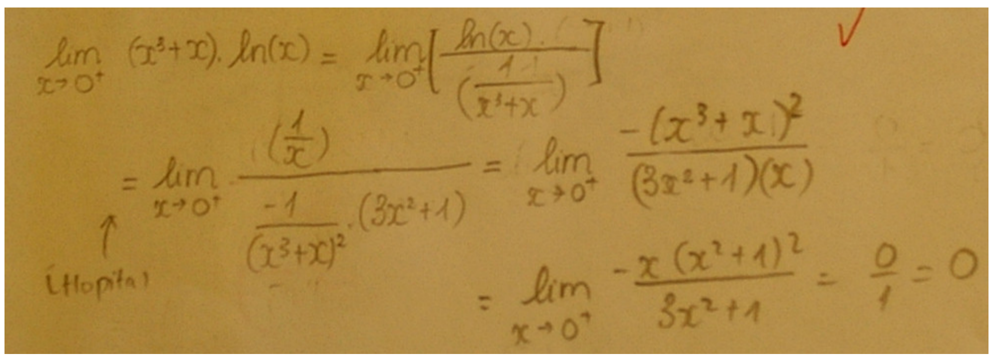
sidebar:

$$\lim_{x\to\infty} x\ln\left(1-\tfrac{3}{x}\right) = \lim_{x\to\infty}\frac{\ln\left(1-\frac{3}{x}\right)}{\frac{1}{x}} \quad \frac{0}{0}$$

$$= \lim_{x\to\infty}\frac{\frac{1}{1-\frac{3}{x}}\cdot\frac{3}{x^2}}{-\frac{1}{x^2}} = \lim_{x\to\infty}\frac{-3x^2}{x^2-3x} = \lim_{x\to\infty}\frac{-6x}{2x-3} = \lim_{x\to\infty}\frac{-6}{2} = -3$$

Thanks to Luke Goodman

1. (4 pts) Compute the following: $\displaystyle\lim_{x\to 0^+}(x^3+x)\ln(x)$.



$$\lim_{x\to 0^+}(x^3+x).\ln(x) = \lim_{x\to 0^+}\left[\frac{\ln(x)}{\left(\frac{1}{x^3+x}\right)}\right]$$

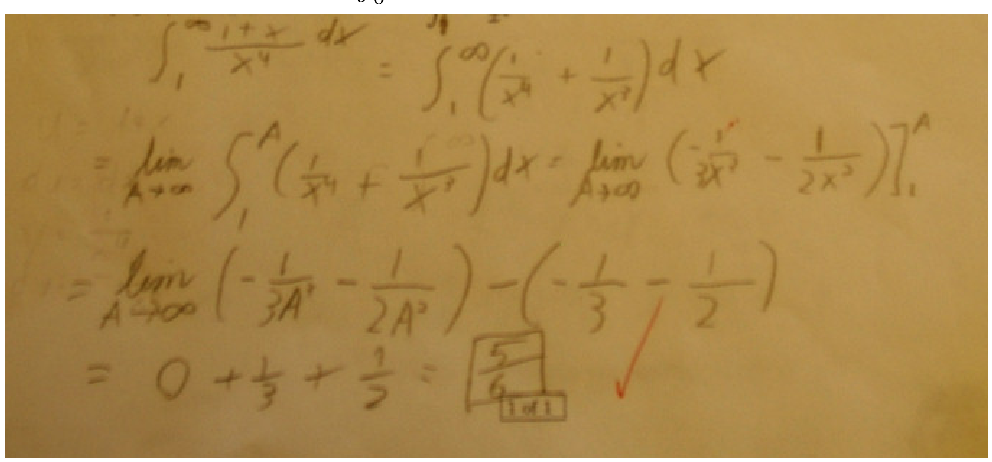
$$= \lim_{x\to 0^+}\frac{\left(\frac{1}{x}\right)}{\frac{-1}{(x^3+x)^2}.(3x^2+1)} = \lim_{x\to 0^+}\frac{-(x^3+x)^2}{(3x^2+1)(x)}$$

L'Hopital

$$= \lim_{x\to 0^+} -\frac{x(x^2+1)^2}{3x^2+1} = \frac{0}{1} = 0$$

Thanks to Ahn Do

3. (2 pts) Compute the following: $\displaystyle\int_0^\infty \frac{1+x}{x^4}\,dx$.



$$\int_1^\infty \frac{1+x}{x^4}\,dx = \int_1^\infty\left(\frac{1}{x^4}+\frac{1}{x^3}\right)dx$$

$$= \lim_{A\to\infty}\int_1^A\left(\frac{1}{x^4}+\frac{1}{x^3}\right)dx = \lim_{A\to\infty}\left(-\frac{1}{3x^3}-\frac{1}{2x^2}\right)\Big]_1^A$$

$$= \lim_{A\to\infty}\left(-\frac{1}{3A^3}-\frac{1}{2A^2}\right)-\left(-\frac{1}{3}-\frac{1}{2}\right)$$

$$= 0+\frac{1}{3}+\frac{1}{2} = \boxed{\frac{5}{6}}$$

Thanks to Benjamin Carlson