Exercises for Lectures 7

These exercises are include exclusively problems on the inclusion exclusion principle:

Exercises

1. In a freshman class of size 586, 226 take calculus, 201 take physics, and 206 take English; 106 take Calculus and Physics. 121 take Physics and English; 96 take calculus and English; and 76 freshmen take all three of these classes. How many Freshmen take none of these courses?

2. Among all permutations of $\{1, 2, 3, 4, 5\}$, how many have 1 in the first position? How many have 2 in the second position? How many have all of 1, 2 and 3 in the correct position. How many have none of the three in the correct position?

3. Answer the same question as the previous for permutations of n numbers.

4. How many strings of six characters either start and end with the same letter, such as "ANDREA", or read the same forwards and backwards, such as "YTAATY", or contain only letters from the set $\{A, E, I, O, U\}$, such as "OOAIUO".

5. Compute, using inclusion/exclusion, how many numbers from 1 to 1000 are either even, or are between 100 and 200, or end in a 5 or a 6.

6. There is an office building with 100 floors. Each floor has 100 offices on the south side facing the sea, and 100 offices on the north side facing the mountains. There are three night watchmen who each check half the offices and make sure that all is secure and that the lights are turned out. Alice checks all the offices on the north side on even floors and those on the south side on odd floors. Bob checks the odd numbered offices on all the floors and both sides. Chuck checks all the offices on the top 50 floors. In the worst case, how many offices have their lights left on? Use inclusion/exclusion.

7. Consider the set of numbers $A = \{1, 2, \ldots, 100,000\}$. How many numbers in A are either even, evenly divisible by 5, or evenly divisible by 7?

8. Consider strings of 12 lowercase letters, such as $aksdjmnuuyio$. How many strings either are a repetition of 2 strings of 6, such as $aksdjmaksdjm$, or a repetition of three strings of 4, such as $akszakszaksz$, or read the same forwards as backwards.

9. Consider the 26^{4n} strings of lowercase letters $a, \ldots, z$ of length $4n$ for a fixed n. How many strings either read the same backwards or forwards, or have repeat after the first $2n$. Let this number be $f(n)$. Compute
$$\lim_{n \to \infty} \frac{f(n)}{26^{4n}}$$