Exercises for Lectures 5

For these exercises, as in class, if A is a set then $\mathcal{P}(A)$ denotes the set of all subsets of A, and $\mathcal{P}_k(A)$ is the set of all subsets of cardinality k.

1. Suppose X is a set with the property that $X \subset A$ for every set A. Show that $X = \emptyset$.

2. Suppose X is a set with the property that $X \cup A \subset A$ for every set A. Show that $X = \emptyset$.

3. Suppose X is a set with the property that $X \subset (X \cap A)$ for all sets A. Show that $X = \emptyset$.

4. Let $A = \{1, 2, 3, 4, 5\}$, let $B = \mathcal{P}(A)$, and $C = \mathcal{P}(C)$. Compute $B \cap C$.

5. Suppose A is a finite set and $\mathcal{P}(A) \subset A$. What can you say about A?

6. Let $A = \{1, 2, 3, 4, 5\}$, let $B = \mathcal{P}(A)$, and $C = \mathcal{P}(C)$. How many subsets of C have? Give three of them.

7. Let $A = \{1, 2, 3, 4, 5\}$, let $B = \mathcal{P}(A)$. How many subsets of 3 elements does A have? How many subsets of 3 elements does B have?

8. Suppose you have a set A containing the 5 vowels, $A = \{a, e, i, o, u\}$. Using the bit vectors, list the 32 subsets of A in lexicographic order.

9. Suppose you have a set B containing the 26 letters, $B = \{a, \ldots, z\}$. Using the usual ordering on the letters, what is the first subset of B in lexicographical order? What is the 128'th subset? What is the 100'th subset?

10. Suppose $A = \{1, 2, 3, 4, 5\}$. Considering all the subsets of A having 3 elements, which is largest in lexicographic order, and which is the smallest? What are their bit vectors?