

Exercises for Lectures 3 and 4

1. Consider the game of chess. What are some sets which help you analyze the game?
2. Which of the following are appropriate to represent by sets: “Members of Congress”, “Politicians”, “Legislators”, “Honest Members of Congress”, “Important Members of Congress”, “Male Members of Congress”, “Members of Congress no more than 5 feet tall”,
3. Which of the following are appropriate to represent by sets: “prime numbers”, “your favorite number”, “real numbers which round down to primes”, “the ages of all your pets”, “numbers which can be represented as the sum, difference, product, or quotient of two prime numbers”, “the ages of all your friends”, “those prime numbers which you friends can correctly identify as prime in 1 second”.
4. Let M be the set of cars produced in year in the United States. What is the complement of that set?
5. Let $A = \{1, 3, 5, 7\}$ and $C = \{2, 5, 7, 8, 9, 10\}$. Compute $A \cup C$, $A \cap C$, $(A \cup B) \cap (A \cap B)$. Using A , C , and \cap and \cup alone, how many sets can be described? Can you prove this?
6. Show that if $A \subset B$ then $A \cup B = B$.
7. Show that if $A \subset B$ then $A \cap B = A$.
8. Let $A = \{1, 3, 5, 7\}$ and $C = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ as above. For how many sets B is it true that $(A \cap C) \cup B = C$?
9. Let the universe consist of all positive integers, and let

$$A_p = \{n \mid n \text{ is divisible by } p\}.$$

Find a nonempty set C such that

$$(\{1\} \cup A_2 \cup A_3 \cup A_5 \cup A_7 \cup A_{11} \cup A_{13} \cup A_{17} \cup A_{19}) \cap C = \emptyset.$$

10. Let A and B be sets with universe \mathcal{U} . Prove $(A \cup B)^c = A^c \cap B^c$
11. Let A and B be sets with universe \mathcal{U} . Prove $(A \cap B)^c = A^c \cup B^c$ in two ways. First directly by showing separately that $(A \cap B)^c \subset A^c \cup B^c$ and $(A \cap B)^c \supset A^c \cup B^c$; second by using the result of the previous exercise.