CS5003 Foundations of C.S. Spring, 2020

Quiz 01000

PRINT NAME: _____

$\mathcal{SIGN}$: _____

1. (7 **pts**) Prove carefully by induction that for all natural numbers n, with $n \geq 2$, The number $n^3 - n$ is evenly divisible by 3.

Use the back if necessary.

♣ *Base Case:* $n = 2$. *We have to show that* $2^3 - 2$ *is divisible by* 3, *which it is since* $2^3 - 2 = 8 - 2 = 6 = 2 \cdot 3$.

Inductive Step: Let $n \geq 2$ *be given, and suppose* $n^3 - n$ *is divisible by* 3, *that is,* $n^3 - n = 3k$ *for some* $k \geq 0$. *We have to check the divisibility of* $(n+1)^3 - (n+1)$.

We compute $(n+1)^3-(n+1) = (n^3+3n^2+3n+1)-(n+1) = n^3+3n^2+3n+1-n-1 = (n^3 - n) + 3n^2 + 3n = 3k + 3n^2 + 3n = 3(k + n^2 + n)$ *by the inductive hypothesis. So* $(n+1)^3 - (n+1)$ *is divisible by* 3, *as required.*

Therefore, the result is true for all $n \geq 2$ *by induction.* ♣

2. (3 **pts**) Suppose for each $n \geq 0$ that P_n is a statement, which is either TRUE or FALSE.

Suppose $P_n \Rightarrow P_{n+5}$ for all $n \geq 0$. Suppose also that P_{555} is TRUE.

For each of the following, place a T if it must be true, and F if it must be false, and X if it cannot be concluded from the given information.

___ P_{10} is FALSE.

___ P_{100} is TRUE.

___ P_{1000} is TRUE.

♣ *Taking* $n = 555$ *as the base case, every index beyond* 555 *which ends in a* 5 *or a* 0 *is true by induction.*

Individual Indices before 555 *might or might not yield true statements, but if any are true, all larger ones must also yield true statements.*

So first and second are both X, *last is* T. ♣