Ma2201/CS2022 | Foundations of C.S. | Spring, 2020
Quiz 0011

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (7 **pts**) Let R be a relation on the integer points of the plane by setting $(n,m)R(n',m')$ if $n - m' = n' - m$.

Prove that R is an equivalence relation. Find the equivalence class of $(5,5)$.

♣ *Reflexivity: Let (n,m) be an element of $\mathbb{N}\times\mathbb{N}$. Reflexivity requires us to show (n,m) is related to itself, or $n-m=n-m$, which is true.*

Symmetry: Let (n,m), (n',m') an elements, so $((n,m),(n',m'))\in R$ and $n-m' = n'-m$. To show symmetry we have to show $((n',m'),(n,m))\in R$ or $n'-m=n-m'$, which is the same equality above.

Transitivity. Suppose now that $((n,m),(n',m'))\in R$ and $((n',m'),(n'',m''))\in R$, so $n-m'=n'-m$ and $n'-m''=n''-m'$. Adding the equations gives $(n-m')+(n'-m'') = (n'-m)+(n''-m')$, and all the single-prime terms cancel out giving: $n-m''=-m+n''$ which says that (n,m) is related to (n'',m''), as required.

So the relation is an equivalence relation.

The equivalence class of $(5,5)$ is all points (n,m) related to $(5,5)$, so $n-5=5-m$, or $n+m=10$, that is, all pairs of numbers which sum to 10. If you want to list them:

$$[(5,5)] = \{(0,10),(1,9),(2,8),(3,7),(4,6),(5,5),(6,4),(7,3),(8,2),(9,1),(10,0)\}$$

♣

2. (3 **pts**) Draw the diagram of any relation on 5 elements which is symmetric and transitive, but not reflexive.

♣ *Let the set be $\{1,2,3,4,5\}$.*

This is a very easy problem. All you have to do is notice that the empty relation ($R=\emptyset$) satisfies these requirements.

Ok, suppose we put in an arrow: $(1,2)$, then by symmetry we have to put $(2,1)$, and then, since they meet at 2 we are forced by transitivity to put in $(1,1)$, and since $(1,2)$ and $(2,1)$ meet at 1, we are forced to put in $(2,2)$ also. So every arrow forces not just the reverse, but the loops at the endpoints. So the arrows give rise to classes just like equivalence classes, and the only Relations which are symmetric and transitive not reflexive are those which have at least one element not related to anything at all. ♣