Ma2201/CS2022
Quiz 0010

Foundations of C.S.

Spring, 2020

PRINT NAME: _____

$\mathcal{SIGN}$: _____

1. (2 **pts**) Define a one=to-one function from $\{1,2,3\}$ to $\mathcal{P}(\{1,2,3\})$.
What does this say about the cardinalities of the sets.

♣ *You can define $f(i) = \{i\}$ for all i This says $|\{1,2,3\}| \leq |\mathcal{P}(\{1,2,3\})|$, which is not surprising.*

2. (4 **pts***) Define an onto function from $\mathbb{N}$ to $\mathbb{Z}$.*
What does this say about the cardinalities of the sets.

♣ The trick is to cover the negative numbers. Here is one way: Given the number n, separate the last digit from the rest by writing $n = 10lk + r$, where $0 \leq r \leq 9$. Then define $f(n) = (-1)^r k$.

Then $f(1776) = 177$ and $f(1775) = 177$.

The function is onto because $f(10n) = n$ for any n.

Lastly, this says the $|\mathbb{Z}| \leq |\mathbb{N}|$.

3. (4 **pts**) For each of the following, label it T for TRUE, F for FALSE, and X if it cannot be determined.

___ $\mathbb{N} \cap \mathcal{P}(\mathbb{N})$ is uncountable.

___ $\mathbb{N} \cup \mathcal{P}(\mathbb{Q})$ is uncountable.

___ $\mathbb{Z} \times \mathbb{Z} \times \mathbb{N}$ us countably infinite.

___ $\mathcal{P}(\mathbb{N} \times \mathbb{Q})$ is countably infinite.

♣ *FIRST: F: Of course the intersection is empty. Even if you don't notice that, $\mathbb{N}$ is countable, and its intersection with something is a subset.*

SECOND: $\mathcal{P}(\mathbb{Q})$ is the power set of an infinite set, so uncountable. So it's union with anything else would still be uncountable.

THIRD: The finite product of countable sets countable.

FOURTH: The power set of an infinite set, so uncountable.