Foundations of C.S.

PRINT NAME: ___________________

$\mathcal{SIGN}$: ___________________

1. (6 **pts**) Prove for sets X, Y and Z that

$$X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z).$$

♣ Proof: We first show that $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$. Let $u \in X \cap (Y \cup Z)$. So $u \in X$ and $u \in Y \cup Z$.

Case 1: $u \in Y$. So we have $u \in X$ and $u \in Y$, so $u \in X \cap Y$, hence $u \in (X \cap Y) \cup (X \cap Z)$.

Case 2: $u \in Z$. So we have $u \in X$ and $u \in Z$, so $u \in X \cap Z$, hence $u \in (X \cap Y) \cup (X \cap Z)$ in this case as well.

So regardless of case, $X \cap (Y \cup Z) \subseteq (X \cap Y) \cup (X \cap Z)$.

We next show that $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$. Let $v \in (X \cap Y) \cup (X \cap Z)$, so there are two cases.

Case 1: $v \in (X \cap Y)$, so $v \in X$ and $v \in Y$. Since $v \in Y$, $v \in Y \cup Z$, so $v \in X \cap (Y \cup C)$.

Case 1: $v \in (X \cap Z)$, so $v \in X$ and $v \in Z$. Since $v \in Z$, $v \in Y \cup Z$, so $v \in X \cap (Y \cup C)$ in this case as well.

Thus $(X \cap Y) \cup (X \cap Z) \subseteq X \cap (Y \cup Z)$.

Thus we have shown $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$. ♣

2. (4 **pts**) Let A and B be sets. For each of the following label it T it it must be true, F if it must be false, and it cannot be determined from the given information.

___ $A \cap B \subseteq \mathcal{P}(A \cup B)$.

___ $A \cap (A \cup B)^c = \emptyset$.

___ $\mathcal{P}(A \cap B) \in \mathcal{P}(A \cup B)$

___ $\emptyset \subseteq A^c \cup B^c$.

♣ *X* *If it was $\in$ and not $\subseteq$, the statement would be always true. For $\in$ it is usually not true, but it might be, say if $A \cap B = \emptyset$.*

F *An element would have to be in A, but not in $A \cup B$, which is impossible. You can also use the distributive law and Demorgan's to analyze it.*

X *If it was $\subseteq$ and not $\in$, the statement would be always true. But as it is, the statement is usually false. However, if $A \cap B = \emptyset$, then $\mathcal{P}(A \cap B) = \{\emptyset\}$, so if $\emptyset \in A \cup B$, the statement is true, and this will happen if $A = \{1, 2, 3\}$ and $B = \{\emptyset\}$. Now their intersection is empty, but their union contains the empty set as an element, so the set containing just the empty set is a subset of $A \cup B$, an so is an element of it's power set. (The was the most difficult point to get on the quiz)*

T *The emptyset is a subset of every set.*

♣