Foundations of C.S.

Spring, 2019

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (6 **pts**) Consider the palindromic on $\{a, b\}$ with grammar

$$G : S \rightarrow aSa \mid bSb \mid cSc \mid a \mid b \mid c \mid \lambda$$

Give a grammar in Chomsky normal form for this language.

You can either convert this one, or write your own from scratch, but it must describe this language.

♣ First step, change to all complex rules to have only variables:

$$\begin{aligned} G : S &\rightarrow ASA \mid BSB \mid CSC \mid a \mid b \mid c \mid \lambda \\ A &\rightarrow a \\ B &\rightarrow b \\ C &\rightarrow c \end{aligned}$$

Now introduce new variables to cut the complex rules to length 2:

$$\begin{aligned} G : S &\rightarrow AX \mid BY \mid CZ \mid a \mid b \mid c \mid \lambda \\ A &\rightarrow a \\ B &\rightarrow b \\ C &\rightarrow c \\ X &\rightarrow SA \\ Y &\rightarrow SB \\ Z &\rightarrow SC \end{aligned}$$

♣

2. (4 **pts**) a) I claim the Chomsky normal form grammar

$$\begin{aligned} G : S &\rightarrow TV \mid AB \mid BC \mid \lambda \\ A &\rightarrow a \\ B &\rightarrow b \\ C &\rightarrow c \\ T &\rightarrow AB \mid AU \\ U &\rightarrow TB \\ V &\rightarrow BW \mid BC \\ W &\rightarrow VC \end{aligned}$$

describes the language $\{w \in \{a, b, c\}^* \mid w = a^i b^j c^k, i + k = j\}$.

Trace through the CYK algorithm to show that $abbbc \notin L(G)$, but $abbbcc \in L(G)$, but (You may use the version described in the book, or the one we illustrated in class.)

♣ Since the algorithm works by sub-words, you don't need to run it twice. Just run it once on *abbbcc*:

$$
\begin{array}{ccccccccccc}
a & & b & & b & & b & & c & & c \\
A & & B & & B & & B & & C & & C \\
 & S,T & & \emptyset & & \emptyset & & S,V & & \emptyset & \\
 & & U & & \emptyset & & \emptyset & & W & & \\
 & & & \emptyset & & \emptyset & & V & & & \\
 & & & & \boxed{\emptyset} & & \emptyset & & & & \\
 & & & & & S & & & & &
\end{array}
$$

The three S's tell you that the only sub-words of *abbbcc* which are in the language: *ab*, *bc* and *abbbcc* itself. In particular, the $\emptyset$ in the box tells you that *abbbc* is not in the language.