Ma2201/CS2022

Quiz 0011

Foundations of C.S.

Spring, 2019

PRINT NAME: ____________________

SIGN: ____________________

1. (**4 pts** Give a recursive definition of the set $\{(n, m) \mid n \in \mathbb{N}, m \in \mathbb{N}, m < n\}$.

♣ Idea: The second coordinate can be any natural number, and given m, the lowest value the first coordinate can have is $m + 1$. Make a first rule generating $(m + 1, m)$, and a second rule which just increments the first coordinate.

BASIS: $(1, 0)$ is in the set.

RECURSIVE STEP: If (p, q) is in the set, $(p + 1, q + 1)$ is in the set. (This alone generates $\{(m + 1, m)\}$.)

If (p, q) is in the set, $(p + 1, q)$ is in the set.

CLOSURE: Every element is generated from the basis after a finite number of applications of the recursive step. ♣

2. (**6 pts**) Prove clearly and carefully by induction that $2 + 4 + 6 + \cdots + 2n = n^2 + n$.

♣ *Proof.* By induction on n.

Base case: For $n = 1$ the statement is $2 \cdot 1 = 1^2 + 1$, which is true. (Since the sum starts with 2, $n = 0$ is not a case. A similar formula with summation notation would work for $n = 0$.

Inductive Step. Suppose $2 + 4 + 6 + \cdots + 2n = n^2 + n$ for some $n \geq 1$. (This is the inductive hypothesis.) We have to compute

$$
\begin{aligned}
2 + 4 + 6 + \cdots + 2n + 2(n + 1) &= (2 + 4 + 6 + \cdots + 2n) + 2(n + 1) \\
&= (n^2 + n) + 2(n + 1) \quad \text{by the inductive hypothesis} \\
&= n^2 + 3n + 2 \\
&= n^2 + 3n + 2 \\
&= (n^2 + 2n + 1) + (n + 1) \\
&= (n + 1)^2 + (n + 1)
\end{aligned}
$$

So the statement is true for $n + 1$, concluding the inductive step.

So the statement is true for all $n \geq 1$ by induction.

Since the second problem was a proof, you were not asked to prove that your answer was correct, but you should be able to do this. ♣