Ma2201/CS2022 Foundations of C.S. Spring, 2019
Quiz 0010

PRINT NAME: ________________

$\mathcal{SIGN}$: ________________

1. (**4 pts**) Let $f : \mathbb{N} \to X$ be one to one.

Label each of the following with T if it must be true, F, it it must be false, and X if there is not enough information given.

___ X is countable.

♣ X: We can conclude from the one-to-one function f that $|\mathbb{N}| < |X|$. So X is an infinite set, but whether it is countable or not cannot be determined from f. ♣

___ X is uncountable.

♣ X: We can conclude from the one-to-one function f that $|\mathbb{N}| < |X|$. So X is an infinite set, but whether it is countable or not cannot be determined from f. ♣

___ $\mathbb{N} \times \{x \in X \mid x = f(k), k \in \mathbb{N}\}$ is countable.

♣ T: f is onto $\{x \in X \mid x = f(k), k \in \mathbb{N}\} \subseteq X$, so $\{x \in X \mid x = f(k), k \in \mathbb{N}\}$ is countable, and the product of two countable sets is countable. ♣

___ There is an onto function $g : X \to \mathbb{N}$.

♣ T: This follows directly, or from the first questions where have have $|X| < |\mathbb{N}|$. ♣

2. (**6 pts**) Let $R \subseteq (\mathbb{Z} \times \mathbb{Z}) \times \mathbb{Z} \times \mathbb{Z})$ be defined by

$$R = \{((p, q), (r, s)) \mid p + s^2 = r + q^2\}.$$

Show that R is an equivalence relation, and find the equivalence class of $(0, 0)$.

♣ We have to show reflexivity, symmetry and transitivity.

Reflexive: Let $(n, m) \in \mathbb{Z} \times \mathbb{Z}$. Then $n + m^2 = n + m^2$, so $((n, m), (n, m)) \in R$.

Symmetric: Let $((p, q), (r, s)) \in R$, so $p + s^2 = r + q^2$. Then $r + q^2 = p + s^2$ and $((r, s), (p, q)) \in R$

Transitive: Let $((p, q), (r, s)) \in R$, and $((r, s), (t, u) \in R$, so $p + s^2 = r + q^2$ and $r + u^2 = t + s^2$. Adding the two equations gives $p + s^2 + r + u^2 = r + q^2 + t + s^2$, and canceling give $p + u^2 = q^2 + t$, so $((p, q), (t, u)) \in R$, and the relation is transitive.

So the relation is an equivalence relation.

Now, the equivalence class of $(0, 0)$ is all pairs (n, m) such that $((0, 0), (n, m)) \in R$, that is, $0 + m^2 = n + 0^2$, so the equivalence class of $(0, 0)$ is $\{(m^2, m) \mid m \in \mathbb{Z}\}$. ♣