Ma2201/CS2022
Quiz 0001

Foundations of C.S.

Spring, 2019

PRINT NAME: ________________

$\mathcal{SIGN}$: ________________

1. Let A, B and C be sets. Use the definition of set equality (that is, the double inclusion method) to show that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Solution

First show $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

Let $x \in A \cap (B \cup C)$. So $x \in A$ and $x \in B \cup C$).
Case 1: If $x \in B$, then $x \in A \cap B$ and $x \in (A \cap B) \cup (A \cap C)$.
Case 2: If $x \in C$, then $x \in A \cap C$ and $x \in (A \cap B) \cup (A \cap C)$ in this case as well.
So, $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

Next we show $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.

Let $y \in (A \cap B) \cup (A \cap C)$, so $y \in A \cap B$ or $y \in A \cap C$.
Case 1: If $y \in A \cap B$, then $x \in A$ and $y \in B$. Since $y \in B$, we have $y \in B \cap C$, so $y \in A \cap (B \cup C)$.
Case 2: If $y \in A \cap C$, then $x \in A$ and $y \in C$. Since $y \in C$, we have $y \in B \cap C$, so $y \in A \cap (B \cup C)$ in this case as well.
Thus $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$.

Therefore,

since $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ and $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$. we have $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

2. Let $D = \{1, 2, 3\}$. Find an onto function $f : \mathcal{P}(D) \to D$.

There are many many examples of this. Here is one. For any subset with exactly one element, $\{n\}$, define $f(\{n\}) = n$. This insures it is onto. Otherwise define $f(X) = 1$. So $f(\emptyset) = f(\{1, 2\}) = f(\{1, 3\}) = f(\{2, 3\}) = f(\{1, 2, 3\}) = 1$ and $f(\{1\}) = 1$, $f(\{2\}) = 2$, and $f(\{3\}) = 3$.