



CS5003
Quiz 1010

Foundations of C.S.

Spring, 2017

PRINT NAME: _____

SIGN: _____

1. (5 points) Convert the following grammar to an equivalent grammar in Chomsky Normal Form.

$$\begin{aligned}G : S &\rightarrow AACB \mid C \\ A &\rightarrow cA \mid Ac \mid a \\ B &\rightarrow cB \mid Bc \mid b \\ C &\rightarrow cbCac \mid \lambda\end{aligned}$$

♣ First we get rid of the λ rules and chain rules. $\text{NULL} = \{S, C\}$, so we convert to

$$\begin{aligned}S &\rightarrow AACB \mid AAB \mid C\lambda \\ A &\rightarrow cA \mid Ac \mid a \\ B &\rightarrow cB \mid Bc \mid b \\ C &\rightarrow cbCac \mid cbac\end{aligned}$$

Then the only non-trivial chain is $\text{CHAIN}(S) = \{S, C\}$, so we convert to

$$\begin{aligned}S &\rightarrow AACB \mid AAB \mid cbCac \mid cbac \mid \lambda \\ A &\rightarrow cA \mid Ac \mid a \\ B &\rightarrow cB \mid Bc \mid b \\ C &\rightarrow cbCac \mid cbac\end{aligned}$$

Lastly, using the usual constant variables, we get

$$\begin{aligned}S &\rightarrow AX_1 \mid AY_1 \mid U_cZ_1 \mid U_cW_1 \mid \lambda \\ A &\rightarrow U_cA \mid AU_c \mid a \\ B &\rightarrow U_cB \mid BU_c \mid b \\ C &\rightarrow U_cZ_1 \mid U_cW_1 \\ X_1 &\rightarrow AX_2 \quad X_2 \rightarrow CB \\ Y_1 &\rightarrow AB \\ Z_1 &\rightarrow U_bZ_2 \quad Z_2 \rightarrow CZ_3 \quad Z_3 \rightarrow U_aU_c \\ W_1 &\rightarrow U_bW_2 \quad W_2 \rightarrow U_aU_c \\ U_a &\rightarrow a \quad U_b \rightarrow b \quad U_c \rightarrow c \quad \clubsuit.\end{aligned}$$

2. (5 points) Construct a grammar in Chomsky Normal Form for the language

$$L = \{w \in \{a, b\}^* \mid n_a(w) + n_b(w) \text{ is even}\}.$$

♣ The best way is to construct first a regular grammar, which necessarily will be almost in the correct form.

We need only three variables, S : start, A : prefix has $n_a(p) + n_b(p)$ even, B : prefix has $n_a(p) + n_b(p)$ odd. If we weren't excluding a recursive start we would only need two variables.

$$\begin{aligned} G : S &\rightarrow aB \mid bB \mid \lambda \\ A &\rightarrow aB \mid bB \\ B &\rightarrow aA \mid bA \mid a \mid b \end{aligned}$$

Now the Chomsky Normal Form is

$$\begin{aligned} G : S &\rightarrow U_a B \mid U_b B \mid \lambda \\ A &\rightarrow U_a B \mid U_b B \\ B &\rightarrow U_a A \mid U_b A \mid a \mid b \\ U_a &\rightarrow a \\ U_b &\rightarrow b \quad \clubsuit. \end{aligned}$$