CS5003
Quiz 1000

Foundations of C.S.

Spring, 2017

PRINT NAME: ____________________
SIGN: ____________________

1. (4 points) Construct a **regular** grammar for the language on $\Sigma = \{a, b, c\}$ consisting of all strings w on $\Sigma = \{a, b, c\}$ with $n_a(w) + n_b(w) = 3k$, with $k \in \mathbb{N}$.

♣ For sentential forms, we need only consider whether the prefix w satisfies, $n_a(w) + n_b(w) = 3k$ or $n_a(w) + n_b(w) = 3k + 1$ or $n_a(w) + n_b(w) = 3k + 2$ and we chose varriables S, A and B to mark those possibilities.

$$\begin{aligned} G : S &\to \lambda \mid aA \mid bA \mid cS \\ A &\to aB \mid bB \mid cA \\ B &\to aS \mid bS \mid cS \mid a \mid b \end{aligned}$$

2. (6 points) Let G be the grammar

$$G : S \to aSbb \mid aaSb \mid \lambda$$

a) Show that G is ambiguous.

♣ One source of the ambiguity, is that the two rules applied in any order yield the same result.

$$\begin{aligned} S &\Rightarrow aSbb \Rightarrow aaaSbbb \Rightarrow aaabbb \\ S &\Rightarrow aaSb \Rightarrow aaaSbbb \Rightarrow aaabbb \end{aligned}$$

are two different right derivations of $aaabbb$.

Their trees are distinct, which also shows ambiguity.

```
     S            S
    /|\\         //|\
   a S bb      a a S b
    //|\          /|\\
  a a S b        a S bb
      |            |
      λ            λ
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b) Give an unambiguous grammar with the same language.

♣ The ordering is the only source of the ambiguity. The language is $a^{i+2j}b^{2i+j}$, with i applications of the first rule and j applications of the second rule. If $a^n b^m$ is in the language, then solving $n = i + 2j$ and $m = 2j + i$ gives $i = (2m - n)/3$ and $j = (2n - m)/3$, so there is at most one pair i and j that work. An unambiguous grammar forces all rules of one type to happen first:

$$\begin{aligned} G : S &\to aSbb \mid aaAb \mid aab\lambda \\ A &\to aaAb \mid aab \end{aligned}$$ ♣