



1. (4 points) Construct a grammar for the language on  $\Sigma = \{a, b, c\}$  consisting of all strings with either an even number of letters, or be a string of the form  $a^{2k}b^{2k+1}c^3$ ,  $k \geq 0$ .

♣ We use  $A$  and  $B$  to split the types. From  $A$  will derive all strings with an even numbers of characters. From  $B$  will derive the others.

$$\begin{aligned} G : S &\rightarrow A \mid Bbc^3 \\ A &\rightarrow aaA \mid abA \mid acA \mid baA \mid bbA \mid bcA \mid caA \mid cbA \mid ccA \mid \lambda \\ B &\rightarrow a^2Ab^2 \mid \lambda \quad \clubsuit \end{aligned}$$

2. (6 points) Let  $G$  be the grammar given by

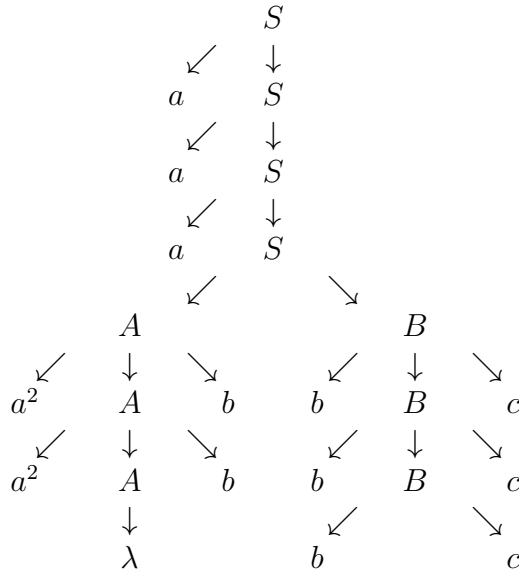
$$\begin{aligned} G : S &\rightarrow aS \mid AB \\ A &\rightarrow a^2Ab \mid \lambda \\ B &\rightarrow bBc \mid bc \end{aligned}$$

a) Give a derivation sequence for  $a^7b^5c^3$

♣ Consider rewriting  $a^7b^5c^3 = a^3(a^4b^2)(b^3c^3)$  and the derivation sequence is

$$S \xrightarrow{3} a^3S \Rightarrow a^3AB \xrightarrow{2} a^3a^4Ab^2B \Rightarrow a^3a^4b^2B \xrightarrow{2} a^3a^4b^2b^2Bc^2 \Rightarrow a^3a^4b^2b^2bcc^2 \quad \clubsuit$$

b) On the back of this paper draw the derivation tree of your derivation of part a.



c) Give a set theoretic definition of the language.

♣ From  $B$  derives  $b^{k+1}c^{k+1}$ ,  $k \geq 0$ . From  $A$  derives  $a^{2j}b^j$ ,  $j \geq 0$ . From  $S$  derives  $a^iAB$ ,  $i \geq 0$ . So the language is

$$\{a^{i+2j}b^{j+k+1}c^{k+1} \mid i \geq 0, j \geq 0, k \geq 0\}, \quad \clubsuit$$