



1. (2 points) Let $\Sigma = \{a, b, c\}$. Give a regular expression for the set of strings in which the substring aaa occurs exactly once.

♣ Before the occurrence of the only aaa , the a 's have power at most 2 followed by at least one other letter, so $(a \cup a^2)(b \cup c)^+$. There may be an initial string of non- a letters. After the occurrence of aaa the string following is built with the reverse consideration, so

$$(b \cup c)^*((a \cup a^2)(b \cup c)^+)^*aaa((b \cup c)^+(a \cup a^2))^*(b \cup c)^* \quad \clubsuit$$

2. (3 points) Let $\Sigma = \{a, b, c\}$. Give a regular expression for the of strings in which the substring $aaaa$ does not occur.

♣ The initial strings of the first problem is a hint for this one. Consider first those strings we want which start and end in a , then $(a \cup a^2 \cup a^3)((b \cup c)^+(a \cup a^2 \cup a^3))^*$ works. Then we can tack on possible beginnings and endings, and don't forget the empty string:

$$(b \cup c)^*((a \cup a^2 \cup a^3)((b \cup c)^+(a \cup a^2 \cup a^3))^* \cup \lambda)(b \cup c)^* \quad \clubsuit$$

3. (2 points) The language of decimal expressions for rational numbers. (In case you have forgotten, every such expression is either a finite decimal, or a repeating decimal, with the repeating string signalled by a bar over the top.)

$$\Sigma = \{., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, \bar{0}, \bar{1}, \bar{2}, \bar{3}, \bar{4}, \bar{5}, \bar{6}, \bar{7}, \bar{8}, \bar{9}\}$$

Your language should account for strings like 15, .25, 15.02, 215.1234566500 and repeating decimals like 2.025 $\bar{227}$ (You may use grep notation, so $[1..5]$ for $(1 \cup 2 \cup 3 \cup 4 \cup 5)$ and the same for the barred digits.)

♣ The integer part is $[0..9]^*$ and there may be a decimal part consisting of a decimal point, and a finite collection of digits followed perhaps by a finite string of barred digits which indicates a repeating string. $[0..9]^*(\lambda \cup .[0..9]^*[\bar{0}..\bar{9}]^*)$ would be ok except that it allows for just the decimal point. To make sure that something occurs on one or the other side we need something like

$$([0..9]^+(\lambda \cup .[0..9]^*[\bar{0}..\bar{9}]^*)) \cup (.[0..9]^+[\bar{0}..\bar{9}]^*) \cup (.[\bar{0}..\bar{9}]^+) \quad \clubsuit$$

4. (3 points) Let $\Sigma = \{a, b, c\}$. The set of all strings of odd length which contain exactly 2 b 's.

♣ Such a string will be of the form $w_1bw_2bw_3$, with w_i strings on $\{b, c\}$ and an even number of them are of odd length. So the expression is the union of the four types:

$$\begin{aligned} & ((a \cup c)^2)^*b((a \cup c)^2)^*b((a \cup c)^2)^* \cup ((a \cup c)^2)^*(a \cup c)b((a \cup c)^2)^*(a \cup c)b((a \cup c)^2)^* \\ & \cup ((a \cup c)^2)^*b((a \cup c)^2)^*(a \cup c)b((a \cup c)^2)^*(a \cup c) \\ & \cup ((a \cup c)^2)^*(a \cup c)b((a \cup c)^2)^*b((a \cup c)^2)^*(a \cup c) \end{aligned} \quad \clubsuit$$