CS5003 | Foundations of C.S. | Spring, 2017

Quiz 0100

PRINT NAME: ____________________

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1. Give a (4 points) Let $\Sigma = \{a, b, c\}$ and let L be the language definite recursively by

BASIS: $\lambda \in L$

RECURSIVE STEP: If $w \in L$ then $wa^3 \in L$, $wa^2b \in L$, and $awbwbw \in L$.

CLOSURE: The elements of L are obtained from the basis after a finite number of applications of the recursive step.

Prove that every string in L has length divisible by 3.

♣ Proof by induction on the number of recursive steps applied. Equivalently, we prove that every string in L_n has length divisible by 3 for all n, by induction on n.

Base case: $n = 0$. L_0 is the base case so $w = \lambda$, the length$(w) = 0 = 3 \cdot 0$, so the result is true in the base case.

Inductive Step: Suppose the result is true for all elements of L_n. Let $w' \in L_{n+1}$, so $w' \in \{wa^3, wa^2b, awbwbw\}$ for some $w \in L_n$. By the induction hypothesis, length$(w) = 3k$ for some k. We easily compute length$(wa^3) =$ length$(wa^2b) = 3k+3$ and length$(awbwbw) = 3(3k) + 3$, all of which are divisible by 3, so w has length divisible by 3 and the induction step is complete.

Since the base case and the inductive steps are true, the statement is true for all $n \geq 0$ by induction. ♣

2. (6 points) Let $\Sigma = \{a, b, c\}$ Give a regular expression for each of the following:

a) The set of set of all strings of length 7 which start with a or b and end with b or c.

♣ The first and last characters are represented by $(a \cup b)$ and $(b \cup c)$ respectively, and the middle segment is of length 5, so by concatenation the regular expression is

$$(a \cup b)(a \cup b \cup c)^5(b \cup c). \qquad ♣$$

b) The set of all strings which contain either the substring abc or the substring cba.

♣ (OR means either or or both!)

We can just do each of these conditions separately, and use the union.

The set of all strings which contain the substring abc is represented by $(a \cup b \cup c)^*abc(a \cup b \cup c)^*$

The set of all strings which contain the substring cba is represented by $(a \cup b \cup c)^*cba(a \cup b \cup c)^*$

Altogether the language is $((a \cup b \cup c)^*abc(a \cup b \cup c)^*) \cup ((a \cup b \cup c)^*cba(a \cup b \cup c)^*)$ or $(a \cup b \cup c)^*(abc \cup cba)(a \cup b \cup c)^*$ ♣

c) The set of all strings which contain both the substrings abc and the substring cba.

♣ AND signals the intersection we so have to think carefully. Either the special substrings intersect or not. If they intersect, they can have at most and a or c in common, so either $abcba$ or $cbabc$. If they don't intersect, there may be a string of any type in between, so $abc(a \cup b \cup c)^*cba$, or $cba(a \cup b \cup c)^*abc$, depending on which is first. So allowing for the beginning and end it must match

$$(a \cup b \cup c)^*(abcba \cup cbabc \cup abc(a \cup b \cup c)^*cba \cup cba(a \cup b \cup c)^*abc)(a \cup b \cup c)^* \qquad ♣.$$