CS5003 Foundations of C.S. Spring, 2017

Quiz 0011

PRINT NAME: _____

$\mathcal{SIGN}$: _____

1. (6 points) Prove by induction on n that

$$1 + 6\sum_{k=0}^{n} 7^k = 7^{n+1}$$

♣ Proof by induction on n.

Base case: $n = 0$. In this case $1 + 6\sum_{k=0}^{0} 7^k = 1 + 6(7^0) = 1 + 6 = 7 = 7^{0+1}$, so the base case is true.

Inductive Step:Suppose the formula $1 + 6\sum_{k=0}^{n} 7^k = 7^{n+1}$ is true for some particular n. We have to show it for $n + 1$. The left hand side is

$$\begin{aligned} 1 + 6\sum_{k=0}^{n+1} 7^k &= \left[1 + 6\sum_{k=0}^{n} 7^k\right] + 6(7^{n+1}) \\ &= 7^{n+1} + 6(7^{n+1}) \qquad \text{By the inductive hypothesis} \\ &= (1+6)7^{n+1} = 7^{n+2} = 7^{(n+1)+1} \end{aligned}$$

which is the right hand side for $n + 1$ as required.

Since the base case is true, and the inductive step, the equation is true for all $n \geq 0$ by induction. ♣

2. (4 points) Consider the recursive definition of the language L over alphabet $\Sigma = \{a, b\}$.

BASIS: $\lambda \in L$.

RECURSIVE STEP: If $w \in L$ then $wa^3 \in L$, $wa^2b \in L$, and $abwb \in L$.

CLOSURE: The elements of L are obtained from the basis after a finite number of applications of the recursive step.

a) $L_0 = \{\lambda\}$. Compute L_2.

♣ $L_1 = \{\lambda, a^3, a^2babb\}$

$L_2 = \{\lambda, a^3, a^2b, abb, a^3a^3, a^3a^2b, aba^3b, a^2ba^3, a^2ba^2b, aba^2bb, abba^3, abba^2b, ababbb\}$

$= \{\lambda, a^3, a^2b, ab^2, a^6, a^5b, aba^3b, a^2ba^3, a^2ba^2b, aba^2b^2, ab^2a^3, ab^2a^2b, aba^3\}$

♣

b) Is $abababababaaaaaaaaaaaaababaaaaaaaaaaaaa \in L$? Why are why not?

♣ It is not in the language. We can work backwards. The only recursive step yielding words ending in a is wa^3, so we would need that $abababababaaaaaaaaaaaaabab \in L$.

The two recursive steps that yield words ending in b are wa^2b, and $abwb$. The first could not have been used, since it doesn't end in a^2b, so it must be the second, and we would need $abababababaaaaaaaaaaaaaba \in L$. But this word ends in a but not in a^3, so it is the result of none of the recursive steps, so is not in L. So our original word is not in L.

— A much faster argument is: it is easy to see that all words in L have length divisible by 3, since the base case is a word of length 0, and each recursive step adds 3 letters. The word in question has length 37, so is not in L. ♣