CS5003
Quiz 0001

Foundations of C.S.

Spring, 2017

PRINT NAME: ____________________
$\mathcal{SIGN}$: ____________________

1. (6 points) Let A and B be sets. Show using the double implication method that $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

Solution

♣

First show $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$

Let $x \in \overline{A \cup B}$, *so* $x \notin A \cup B$. *Hence* $x \notin A$ *and* $x \notin B$. *Therefor* $x \in \overline{A}$ *and* $x \in \overline{B}$, *hence* $x \in \overline{A} \cap \overline{B}$, *as required.*

Next show $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$

Let $y \in \overline{A} \cap \overline{B}$. *So* $y \in \overline{A}$ *and* $y \in \overline{B}$. *So* $y \notin A$ *and* $y \notin B$. *Hence* $y \notin A \cup B$, *so* $y \in \overline{A \cup B}$, *as required.*

Since $\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$ *and* $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$ *we have* $\overline{A \cup B} = \overline{A} \cap \overline{B}$ ♣

2. (4 pts) Let $X = \{\emptyset, \{2, 3\}\}$ and let $Y = \mathcal{P}(X)$. Label each of the following TRUE or FALSE.

___ $X \cap Y = \emptyset$.

♣ False – $X \cap Y = \{\emptyset\}$

___ There are at least two one-to-one functions from Y to X.

♣ False – $|X| = 2 < |Y| = 2^2 = 4$ so there are no one-to-one functions from Y to X.

___ $X \subseteq Y$.

♣ False – $X \subseteq X$, so $X \in Y$, and $\{X\} \subseteq Y$.

___ There are at least two onto functions from X to Y.

♣ False – $|X| < |Y|$, so there are no onto functions from X to Y.