Summary of Lectures 23 and 24

We discussed recursion and, in particular, the Fibonacci Numbers

$$f_0 = 0, f_1 = 1, f_{n+1} = f_n + f_{n-1}.$$

We showed an upper bound and a lower bound by induction.

We introduced "strong induction", and showed how it is important not to neglect checking the base case carefully.

We proved by induction the crazy formula

$$f_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

showed how the binomial theorem makes it plausible, and discussed it's implications.

Exercises for Lectures 23 and 24

[**All just for fun. This is not on the final.**]

1. We showed that 2^n is an upper bound by (strong) induction,

   $$f_n \leq 2^n.$$

   Show that $(1.9)^n$ is also an upper bound using exactly the same method.

2. Try also $(1.8)^n$, $(1.7)^n$, until the method fails.

3. Find the smallest constant c such that c^n is an upper bound of the Fibonacci sequence.

   (Maybe he wasn't so crazy!)

4. Fibonacci studied very conservative rabbits which only had 1 pair of rabbits in a litter.

   It is recorded that a pair of *oryctolagus cuniculus domesticus* had a litter of 12 pairs!

   Write a recursive system for this breed of rabbits, assume maximum litter size.

   Find the best upper bound for its growth that you can.