Summary of Lectures 17 and 18

Formal logic concerns *statements*.

A statement is either TRUE (1) or FALSE (0).

Statements can be formed from $\wedge$ (AND), $\vee$ (OR) and $\neg$ (NOT).

We stated the distributive laws:

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \qquad p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

and Demorgan's laws:

$$\neg(p \vee q) = (\neg p) \wedge (\neg q) \qquad \neg(p \wedge q) = (\neg p) \vee (\neg q)$$

We discussed logical implication: $p \Rightarrow q$, IF p THEN q, or p IMPLIES q.

$$(p \Rightarrow q) = (q \vee \neg p)$$

IMPORTANT: We showed the double implication method for showing $p \Leftrightarrow q$ by showing separately that $p \Rightarrow q$ and $q \Rightarrow p$

Exercises for Lectures 17 and 18

1. Suppose p is TRUE and q is FALSE, and r is a statement. Label each of the following as true or false, or undecidable:

   ____ $(p \wedge q \wedge r)$

   ____ $(p \vee q \vee r)$

   ____ $p \wedge \neg(q \vee \neg q)$.

   ____ $p \wedge (p \vee q) \wedge (p \vee q \vee r)$.

   ____ $p \vee (p \wedge q) \vee (p \wedge q \wedge r)$.

   ____ $p \vee \neg((p \wedge q) \vee \neg(p \wedge q \wedge r))$.

2. Suppose $p \wedge (q \vee (p \wedge q))$ is TRUE. What can you conclude about the truth of p and q?

3. Use the double implication method to show that

$$p \vee (q \wedge r) \Longleftrightarrow (p \vee q) \wedge (p \vee r).$$

4. Use the double implication method to show that

$$p \wedge (q \vee r) \Longleftrightarrow (p \wedge q) \vee (p \wedge r).$$

5. Use the double implication method to show that

$$\neg(p \vee q) \Longleftrightarrow (\neg p \wedge \neg q).$$

6. Use the double implication method to show that

$$\neg(p \wedge q) \Longleftrightarrow (\neg p \vee \neg q).$$

7. Show that $[(p \Rightarrow q) \Rightarrow r] \Longrightarrow [p \Rightarrow (q \Rightarrow r)]$.

8. Show that $[(p \Rightarrow q) \Rightarrow r] \Longrightarrow [p \Rightarrow (q \Rightarrow r)]$.

9. Just for fun. Show that $[(p \wedge \neg r) \vee (q \wedge \neg p) \vee (r \wedge \neg q)] \Longleftrightarrow [(p \wedge \neg q) \vee (q \wedge \neg r) \vee (r \wedge \neg p)]$.

   [Hint: You can use the double implication method, or you can use the distributive law to show that both are equivalent to $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$, that is, at least one is true, and at least one is false.]