Discrete Mathematics D Term 2019

Lectures 09 and 10

Summary

We discussed the principle of inclusion/exclusion, and how to justify it using the alternating sum along the rows of Pascal's triangle.

We wrote out the consequence out carefully for two, three and four sets, as well as a general formula:

Let I be a finite set, and let $\{A_i \mid i \in I\}$ be a collection of sets, then

$$\left|\bigcup_{i\in I} A_i\right| = \sum_{J\subseteq I, J\neq\emptyset} (-1)^{|J|-1} \left|\bigcap_{j\in J} A_j\right|$$

We started to examine structure on sets, looking at *relations*

$$R \subseteq X \times Y$$

with several examples, considering different notations, and relational diagrams.

We discussed the special case of a relation corresponding to a permutation of a set.

Exercises for Lectures 09 and 10

In any these problems, it is sufficient to have an algebraic expression for the answer.

1. How many pairs of digits ($\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$) either add to 10, or are equal, or are two apart.

   ♣ *This can be done with inclusion/exclusion.*

   Let A be the set of pairs which add to 10*,*

   $A = \{(1, 9), (2, 8), (3, 7), (4, 6), (5, 5), (6, 4), (7, 3), (8, 2), (9, 1)\}$.

   Let B be the set of pairs which are equal.

   $B = \{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7), (8, 8), (9, 9)\}$.

   Let C be the set of pairs which are two apart.

   $C = \{(0, 2), (1, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 8), (7, 9), (9, 7), (8, 6), (7, 5), (6, 4), (5, 3), (4, 2), (3, 1), (2, 0)\}$.

   We want $|A\cup B\cup C| = |A|+|B|+|C|-|A\cap B|-|A\cap C|-|B\cap C|+|A\cap B\cap C| = 9 + 10 + 16 - 1 - 2 - 0 + 0 = 32$.

   (I wrote out the beginning sets just for fun, but to solve the problem, you only need to know how many are in each set. It is a little more work, but not much more, to do the analogous problem with the numbers $\{0, 1, 2, 3, \ldots, n-1\}$, where you can't just list them all. So add to n, or are equal, or are two apart. Try it.)

   ♣

2. A worker gets every weekend off, a week off to celebrate each full moon, and earns four week's vacation after he was worked 24 weeks. (I think he works in Europe.)

   How many days does he work in 140 weeks?

   ♣ *Hint: A full moon happens every 28 days.* ♣

3. How many many numbers at least 1 million and less than 2 million are divisible by 2, or 5.

4. How many many numbers at least 3 million and less than 9 million are divisible by 2, or 3 or 5.

5. How many many numbers in the set $\{1, 2, 3, \ldots, 60\}$ are divisible by 2 or 3, or 5.

6. Compute all the strings on 5 characters which are either palindromic, start with 3 equal characters, or end with three equal characters.

7. Compute all the strings on 6 characters which are either palindromic, start with 3 equal characters, or end with three equal characters.

8. Compute all the strings on n characters which are either palindromic, start with 3 equal characters, or end with three equal characters.

   Try to write up the result taking into account all possible cases of legal n's.

9. Let $S = \{a, b, c, d, e\}$ and T be the relation defined $\mathcal{P}_1(S) \times \mathcal{P}_4(S)$ defined by $(X, Y) \in T$ if $X \subseteq Y$.

   What is $|T|$?

   Draw a diagram of the relation.

   What is the relation of $|T|$ to your diagram?

10. Let R be the relation defined on the $D \times N$, where D is the set of digits, and X is the set of numbers from 20 to 40, where $(d, n) \in R$ if the digit d occurs in the number n.

    (So $(5, 25) \in R$ and $(2, 25) \in R$, but $(0, 25) \notin R$.

    How many elements are in the relation. (Try inclusion/exclusion)

    Draw the diagram of the relation.

11. Let $S = \{a, b, c, d, e\}$ and R be the relation defined $\mathcal{P}_2(S) \times \mathcal{P}_3(S)$ defined by $(X, Y) \in S$ if $X \subseteq Y$.

    What is $|R|$?

    Draw a diagram of the relation.

    What is the relation of $|R|$ to your diagram?

    ♣ *Misprint fixed.* ♣

12. Consider the permutation (anagram) $lingmofa$ of $flamingo$. Draw both relation diagrams of this permutation.

13. Find an anagram of $flamingos$ whose reduced relation diagram (only 9 points) has only triangles.

    (Hint: Work backwards)