Lectures 21 and 22 – Number Theory II

Summary

This was a short week.

These lectures were on Euclid's Coin problem how to use Euclid's Algorithm to find numbers λ and μ so that

$$\gcd m, n = \lambda n + \mu m.$$

We used this to show that every number is "uniquely" factorable into primes.

Then generalized even and odd with "threven", "throdd", and "thweird" integers, then moved on to the integers modulo n for any n.

We defined addition, subtraction, multiplication, and, if possible, division modulo n.

Exercises on Lectures 21 and 22

1. Billy and Bobby will get a prize if the dolls they knock down sum to exactly 50.

How many different combinations of dolls would get them the prize?

[Hint: The guy in the checkered suit is not a doll.]

2. Find a pair of numbers between 1776 and 2017 whose gcd is 111.
3. Find λ and μ so that $\gcd(1776, 2018) = \lambda \cdot 1776 + \mu \cdot 2018$.
4. Find λ and μ so that $1 = \lambda \cdot 111 + \mu \cdot 1111$ or show that that is impossible.
5. Find λ and μ so that $1 = \lambda \cdot 1111 + \mu \cdot 111111$ or show that that is impossible.
6. Find λ and μ so that $1 = \lambda \cdot 21 + \mu \cdot 121$ or show that that is impossible.
7. Find λ and μ so that $1 = \lambda \cdot 169 + \mu \cdot 144$ or show that that is impossible.
8. Fill in addition and multiplication tables for $\mathbb{Z}_{12}$.
9. $7 \cdot 5$ modulo 12.
10. Compute 2^{10}, 2^{100}, and 2^{1000} modulo 12.
11. Solve $5x = 7$ modulo 12.
12. Fill in addition and multiplication tables for $\mathbb{Z}_{13}$.
13. $7 \cdot 5$ modulo 13.
14. Compute 2^{10}, 2^{100}, and 2^{1000} modulo 13.
15. Solve $5x = 7$ modulo 13.