

Lectures 09 and 10

We started formal logic which concerns *statements*. A statement is either TRUE (1) or FALSE (0).

Statements can be formed from \wedge (AND), \vee (OR) and \neg (NOT).

We stated the distributive laws:

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \qquad p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

and Demorgan's laws:

$$\neg(p \vee q) = (\neg p) \wedge (\neg q) \qquad \neg(p \wedge q) = (\neg p) \vee (\neg q)$$

We discussed logical implication: $p \Rightarrow q$, IF p THEN q , or p IMPLIES q .

$$(p \Rightarrow q) = (q \vee \neg p)$$

Exercises for Lectures 9 and 10

1. Suppose p is the statement $x < 5$, q is the statement $x < -5$ and r is the statement $|x| < 5$.

For each of the following, indicate on a number line those values of x for which the statement is TRUE.

- (a) $p \wedge q \wedge r$
- (b) $p \vee q \vee r$
- (c) $\neg p \vee \neg q \vee \neg r$
- (d) $p \wedge (q \vee r)$
- (e) $p \Rightarrow q$
- (f) $q \Rightarrow p$
- (g) $p \Rightarrow (q \Rightarrow r)$
- (h) $(p \Rightarrow q) \Rightarrow r$

2. Suppose p is TRUE and q is FALSE, and r is a statement. Label each of the following as true or false, or undecidable:

- ___ $(p \wedge q \wedge r)$
- ___ $(p \vee q \vee r)$
- ___ $p \wedge \neg(q \vee \neg q)$.
- ___ $p \wedge (p \vee q) \wedge (p \vee q \vee r)$.
- ___ $p \vee (p \wedge q) \vee (p \wedge q \wedge r)$.
- ___ $p \vee \neg((p \wedge q) \vee \neg(p \wedge q \wedge r))$.

3. Suppose $p \wedge (q \vee (p \wedge q))$ is TRUE. What can you conclude about the truth of p and q ?

4. Use the double implication method to show that

$$p \vee (q \wedge r) \iff (p \vee q) \wedge (p \vee r).$$

5. Use the double implication method to show that

$$p \wedge (q \vee r) \iff (p \wedge q) \vee (p \wedge r).$$

6. Use the double implication method to show that

$$\neg(q \vee r) \iff (\neg p \wedge \neg q).$$

7. Use the double implication method to show that

$$\neg(q \wedge r) \iff (\neg p \vee \neg q).$$

8. Show that $[(p \Rightarrow q) \Rightarrow r] \implies [p \Rightarrow (q \Rightarrow r)]$.

9. Show that $[(p \Rightarrow q) \Rightarrow r] \implies [p \Rightarrow (q \Rightarrow r)]$.

10. Show that $[(p \wedge \neg r) \vee (q \wedge \neg p) \vee (r \vee \neg q)] \iff [(p \wedge \neg q) \vee (q \wedge \neg r) \vee (r \vee \neg p)]$.