

Lectures 07 and 08

Cardinality of Finite Sets:

We showed $|\mathcal{P}(X)| = 2^{|X|}$, $|\mathcal{P}_k(X)| = \binom{|X|}{k} = \frac{|X|!}{k!(|X|-k)!}$, $|X \times Y| = |X| \cdot |Y|$, $|X^c| = |U| - |X|$, $|A \cup B| = |A| + |B| - |A \cap B|$.

Moreover we discussed the arrangement of the numbers $\binom{n}{k}$, the binomial coefficients, in Pascal's triangle, and proved the recursion

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

Also, we showed how the relation $|A \cup B| = |A| + |B| - |A \cap B|$ generalizes into the principle of inclusion exclusion,

$$\left| \bigcup_{i=1}^n A_i \right| = \sum_{I \subseteq N, I \neq \emptyset} (-1)^{|I|+1} \left| \bigcap_{i \in I} A_i \right|, \quad \{1, 2, 3, \dots, n\} = N$$

and illustrated how this is useful in solving problems.

Exercises for Lectures 07 and 08

- Let $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$. Compute the cardinalities of the following sets:
 - $X \times X$
 - $X \times X \times X \times X$
 - $\mathcal{P}(X)$
 - $\mathcal{P}(X) \times X$
 - $\mathcal{P}(X) \cup \mathcal{P}(\mathcal{P}(X))$ (careful)
 - $\mathcal{P}(\mathcal{P}(\mathcal{P}(X)))$
 - $\mathcal{P}(\mathcal{P}(X) \times X)$

(If these were too easy, try the same problems with X replaced with \emptyset .)

- Write down the first 10 rows of Pascal's Triangle. (Try to do it accurately within 60 seconds with no assistance.)
- Find all strings on 5 letters, $\{a, b, c, \dots, z\}$, which have the first three letters vowels, $\{a, e, i, o, u\}$, like *eeopt*.
- Find all strings on 5 letters, $\{a, b, c, \dots, z\}$, which either have the first three letters vowels, $\{a, e, i, o, u\}$ or the last 3 letters vowels.
- Find all strings on 5 letters, $\{a, b, c, \dots, z\}$, which either have the first three letters vowels, $\{a, e, i, o, u\}$ or the last 3 letters vowels, or are palindromes, that is, read the same forwards or backwards.

6. Find all strings on 5 letters, $\{a,b,c,\dots,z\}$, which either have the first three letters vowels, $\{a, e, i, o, u\}$ or the last 3 letters vowels, or are palindromes, or have middle letter w .

Only 6 problems! What an easy course!