Lectures 05 and 06

We discussed sets, elements, the empty set, and the powerset, together with notation and examples.

We discussed operations on sets, union, intersection, complement and cartesian product.

We discussed the double inclusion method of showing set equality.

Exercises for Lectures 05 and 06

1. Discuss whether or not each of these sets is well defined.
   - The set of shoes shoes belonging to WPI professors.
   - The set of hairs on your head.
   - The set of cells in your body.
   - The set of trees on campus.
   - WPI buildings.
2. Discuss whether or not each of these sets is well defined.
   - The set of ages of the members of this class.
   - The set of numbers larger than six.
   - The set of prime numbers which are divisible by six.
   - The set of rational numbers which are complex.
   - The set of all numbers which have been been interesting to some human being.
3. Let $X = \{0, 1, 2\}$

   Write down all elements of $\mathcal{P}(X)$.
4. Let $Y = \{a, b, c, d, e\}$

   Write down all elements of $\mathcal{P}(Y)$.
5. Let $Z = \{0, 1\}$

   Write down all elements of $\mathcal{P}(\mathcal{P}(Z))$.
6. Write down all elements of $\mathcal{P}(\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset))))$. (There are 16 of them)
7. Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 3, 6, 8\}$. Compute $A \cup B$, $A \cap B$, $\mathcal{P}(A \cap B)$, $\mathcal{P}(A) \cap \mathcal{P}(B)$
8. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 9, 10\}$ be the universe from which all the elements are drawn. Let $A = \{5\}$ and $B = \{3, 4, 5, 6\}$.

   Compute $A \cup B$, $A \cap B$, $A \cap B^c$, $A^c \cap B^c$, and $A \cap (B \cap A^c)$.

9. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 9, 10\}$ be the universe from which all the elements are drawn. Let $A = \{5\}$ and $B = \{3, 4, 5, 6\}$. Write down three different expressions involving A, B, $\cup$, $\cap$, and $(-)^c$ which denote the empty set.

10. Let A and B be sets. Prove that $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$ using the double inclusion method.

11. Let A and B be sets. Prove that $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ using the double inclusion method.

12. Let A and B be sets. Prove that $(A \cap B)^c = A^c \cup B^c$.

13. Let A and B be sets. Prove that $(A \cup B)^c = A^c \cap B^c$.