Lectures 21 and 22

Because of Patriot's Day and PPD, these comprised the whole week.

We discussed recursion.

We gave recursive constructions of the factorials and the binomial coefficients.

Solved the Fibonacci recursion

$$f_{n+1} = f_n + f_{n-1} \qquad f_0 = 0, f_1 = 1$$

We showed the binomial theorem

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

We showed how to solve homogeneous linear recursions:

$$A_0 a_n + A_1 a_{n+1} + A_2 a_{n+2} + \ldots A_k a_{n+k} = 0$$

with initial conditions a_0, a_1, $\ldots a_k$, by looking for exponential solutions $a_n = Cr^n$, which give the characteristic equation

$$A_0 + A_1 r + A_2 r^2 + \ldots A_k r^k = 0$$

for r, with k solutions, which can be combined satisfy the initial conditions.

We used this to solve $a_{n+1} = 3a_n - 2a_{n-1}; \quad a_0 = 0, a_1 = 1$.

We described the Towers of Hanoi problem. We showed that for 3 spindles and n disks there are 3^n legal positions by the multiplicative principle.

We showed there is a recursive solution to the problem of how many moves m_n to solve the n-disk problem:

$$m_n = 2m_{n-1} + 1 \qquad m_1 = 1$$

and solved the linear recursion to compute that the solution has $2^n - 1$ moves.

Exercises for Lectures 21 and 22

1. Let $T_n = \sum_{k=0}^{n} k^2$. (Our cannonball numbers if you remember.)

   Write a recursive specification for this sequence. Is it linear? Do not forget the initial conditions.

2. Let $s_k = (2k)!/(2^k k!)$.

   Write a recursive specification for this sequence. Do not forget the initial conditions.

3. Suppose $a_0 = 3$ and for $n \geq 1$ we have $a_{n+1} = 6a_n$.

   Find a formula for a_n.

4. Suppose $a_0 = 3$ and for $n \geq 1$ we have $a_{n+1} = 6a_n - 6$.
   Find a formula for a_n.

5. Suppose $a_0 = 1$ and $a_1 = 2$, and for $n \geq 1$ we have $a_{n+1} = 6a_n - 8a_{n-1}$.
   Find a formula for a_n.

6. Suppose $a_0 = 1$ and $a_1 = 2$, and for $n \geq 1$ we have $a_{n+1} = 6a_n - 8a_{n-1} + 2$.
   Find a formula for a_n.

7. Suppose $a_0 = 5$ and $a_1 = 10$, and for $n \geq 1$ we have $a_{n+1} = 7a_n - 12a_{n-1}$.
   Find a formula for a_n.
   What is $\lim_{n \to \infty} a_n$?

8. According the the Binomial Theorem what is

$$\binom{7}{0} + \binom{7}{1}2 + \binom{7}{2}4 + \binom{7}{3}8 + \binom{7}{4}16 + \binom{7}{5}32 + \binom{7}{6}64 + \binom{7}{7}128$$

9. According the the Binomial Theorem what is

$$\binom{7}{0} - \binom{7}{1}2 + \binom{7}{2}4 - \binom{7}{3}8 + \binom{7}{4}16 - \binom{7}{5}32 + \binom{7}{6}64 - \binom{7}{7}128$$

10. According the the Binomial Theorem what is

$$\binom{7}{0} - \binom{7}{1} + \binom{7}{2} - \binom{7}{3} + \binom{7}{4} - \binom{7}{5} + \binom{7}{6} - \binom{7}{7}$$

11. According the the Binomial Theorem what is

$$\binom{7}{0} - \binom{7}{1}\frac{1}{2} + \binom{7}{2}\frac{1}{4} - \binom{7}{3}\frac{1}{8} + \binom{7}{4}\frac{1}{16} - \binom{7}{5}\frac{1}{32} + \binom{7}{6}\frac{1}{64} - \binom{7}{7}\frac{1}{128}$$