Lectures 13 and 14

We started the topic of number theory.

We introduced division with remainder, and showed how it could be use to change number bases, particularly binary and hexadecimal.

We gave the definition of

prime: $p \mid ab \Rightarrow (p \mid a) \vee (p \mid b)$

irreducible. $p = ab \Rightarrow (a = \pm p) \vee (b = \pm p)$.

We showed the Sieve of Eratosthenes, and discussed the work involved in factoring primes.

We noticed that most work with a number of size n involves working with it's digits.

The number of digits of n in base b is $\lfloor \log_b(n) \rfloor + 1 = \left\lfloor \frac{\ln(n)}{\ln(b)} \right\rfloor + 1$.

We compared $\sqrt{n}$ with $\ln(n)$ and showed $\sqrt{n}$ grows exponentially with respect to $\ln(n)$.

Exercises for Lectures 13 and 14

1. Express 2016 in bases 2, 8, 7, 12, and 16.
2. Find a number whose representations in bases 2, 7, and 13 all ends in two 0's.
3. Find a number whose representations in bases 7, and 11 both end in a 5.
4. Use the sieve of Erotosthenes to find all primes less than 144.

   How many primes did you have to sieve?
5. Use the previous list of primes to check if the following numbers are prime:

   5041

   10013

   12317

   How large a number is your list good for?
6. Order the following functions in terms of growth:

   n^2, $(n+1)^2$, $\sqrt{n}$, $\ln(n)$, $\ln(n^2)$, $(\ln(n))^2$.