Lectures 11 and 12

We did some induction proofs.

We showed that any boolean function can be expressed in terms of $\vee$, $\wedge$ and $\neg$. We did this by introducing the *Disjunctive Normal Form*, a disjunction of conjunctive clauses, and *Conjunctive Normal Form*, e.g.

$$(p \wedge \neg q) \vee (\neg p \wedge \neg q \wedge r \wedge s) \vee (\neg p \wedge \neg q \wedge \neg r) \vee (q \wedge r)$$

a conjunction of disjunctive clauses, e.g.

$$(p \vee q) \wedge (p \vee \neg q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (q \vee r)$$

Finally we used inclusion exclusion to complete computing the cardinality of types of functions with domain and targets finite sets, $f : A \to B$, $|A| = n$, $|B| = m$.

Ordinary: m^n.

One to one ($m > n$): $\dfrac{m!}{(m-n)!}$.

One to one and onto ($m = n$): $m!$.

Onto ($m < n$): $\displaystyle\sum_{k=0}^{m} (-1)^k \binom{m}{k} (m-k)^n$.

Exercises for Lectures 11 and 12

1. Express $p \Rightarrow (q \wedge r)$ into disjunctive normal form.

2. Express $p \Rightarrow (q \Rightarrow r)$ in either conjunctive normal form, or disjunctive normal form.

3. Express $(p \Rightarrow q) \Rightarrow r$ in either conjunctive normal form, or disjunctive normal form.

4. Express $(p \Rightarrow q) \Rightarrow r$ in either conjunctive normal form, or disjunctive normal form.

5. Express $p \vee (q \wedge r) \vee (q \wedge \neg r)$ in conjunctive normal form.

6. Express $p \vee (q \wedge r) \vee (q \wedge \neg r)$ in conjunctive normal form.

7. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6, 7, 8\}$. How many onto functions are there from A to B?

   How many onto functions are there from B to A?

8. Let $C = \{1, 2, 3, 4, 5\}$ How many onto functions are there from $\mathcal{P}_2(A)$ to $\mathcal{P}_3(A)$?

   How many onto functions are there from $\mathcal{P}_2(A)$ to A?