Exercises for Lectures 7 and 8

Cardinality of Finite Sets:

Lectures 7 and 8 we discussed functions on sets, the cardinality of infinite sets and introduced formal logic.

If X and Y are finite sets, the number of functions from X to Y, $f : X \to Y$ is $|Y|^{|X|}$. The number of one to one functions is $\frac{|Y|!}{(|Y| - |X|)!}$.

Two sets have the same cardinality if there is a one-to-one and onto function between them.

A set with the same cardinality as $\mathbb{N}$ is said to be *countable.*

We showed $\mathcal{P}(\mathbb{N})$ is uncountable.

We considered several examples. In general, countable unions and finite products of countable sets are countable. An infinite product of a finite sets is uncountable.

Formal logic concerns *statements.* A statement is either TRUE (1) or FALSE (0).

Statements can be formed from $\wedge$ (AND), $\vee$ (OR) and $\neg$ (NOT).

We stated the distributive laws:

$$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r) \qquad p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$$

and Demorgan's laws:

$$\neg(p \vee q) = (\neg p) \wedge (\neg q) \qquad \neg(p \wedge q) = (\neg p) \vee (\neg q)$$

1. find a one-to-one and onto function from $\mathcal{P}_2(\{1, 2, 3, 4, 5\})$ to to $\mathcal{P}_3(\{1, 2, 3, 4, 5, 6\}$, or show that one does not exist.

2. find an onto function from $\mathcal{P}_2(\{1, 2, 3, 4, 5, 6\})$ to to $\mathcal{P}(\{2, 4, 6\}$, or show that one does not exist.

3. find a one-to-one and onto function from $\mathcal{P}_2(\{1, 2, 3, 4\}) \cup \mathcal{P}_2(\{6, 7, 8, 9, 10\})$ to to $\mathcal{P}(\{1, 2, 3, 4\}$, or show that one does not exist.

4. find a one-to-one and onto function from $\mathcal{P}_2(\{1, 2, 3, 4\}) \times \mathcal{P}_2(\{6, 7, 8, 9, 10\})$ to to $\mathcal{P}(\{1, 2, 3, 4\}$, or show that one does not exist.

5. Is $\mathcal{P}_2(\mathbb{N})$ countable. Why or why not?

6. Is $\mathcal{P}_3(\mathbb{N})$ countable. Why or why not?

7. Is $\mathcal{P}_3(\mathbb{Q})$ countable. Why or why not?

8. Is $\mathcal{P}_3(\mathbb{Q} \times \mathbb{Q})$ countable. Why or why not?

9. Is the set of finite subsets of $\mathbb{N}$ countable? Why or why not?

10. Is the set of finite subsets of $\mathcal{P}(\mathbb{N})$ countable? Why or why not?

11. Suppose p is TRUE and q is FALSE, and r is a statement. Label each of the following as true or false, or undecidable:

    ____ $(p \wedge q \wedge r)$

    ____ $(p \vee q \vee r)$

    ____ $p \wedge \neg(q \vee \neg q)$.

    ____ $p \wedge (p \vee q) \wedge (p \vee q \vee r)$.

    ____ $p \vee (p \wedge q) \vee (p \wedge q \wedge r)$.

    ____ $p \vee \neg((p \wedge q) \vee \neg(p \wedge q \wedge r))$.

12. Suppose $p \wedge (q \vee (p \wedge q))$ is TRUE. What can you conclude about the truth of p and q?