Ma2201/CS2022

Discrete Mathematics

D Term, 2013

Quiz 0101

Solutions

1. (**2 points**) Let X and Y be finite sets. What would allow us to conclude that every one-to-one function with domain set X and target set Y is also onto.

Circle the best answer.

a) $X = Y$. b) $|X| = |Y|$.

c) $|\mathcal{P}(X)| = |\mathcal{P}(Y)|$. d) all of the above

d. For finite sets, any of the first three choices above allow us to conclude $|X| = |Y|$, *from which it follows that every one-to-one function is onto.*

2. (**4 points**) Let $A = \{a, b, c\}$ and $B = \{c, d, e, f\}$, and let C be the set of all functions with domain set A and target set B. Let f be a function with domain $\mathcal{P}(A \cup B)$ and target C.

Circle each of the following which must be true.

a) C is uncountable b) C is onto

c) f is not one-to-one d) f is not onto

$|A| = 3$, $|B| = 4$, $|C| = 4^3 = 64$, *and* $|\mathcal{P}(A \cup B)| = 2^{4+3} = 128$.

a) C is finite, so C is not uncountable, (unless you are really bad at counting)

b) C is a set, not a function, so it is not onto.

c) Since the domain has larger cardinality than the target, the pigeonhole principle says that the function f cannot be onto.

d) Since the target is strictly smaller than the domain, f could be onto.

3. (**4 points**) Let $A = \{a, b, c\}$ and $B = \{c, d, e, f\}$, and let C be the set of all functions with domain set A and target set B, and let D be the set of all functions with domain set B and target A.

Compute the number of one-to-one functions with domain set D and target set C.

Compute the number of onto functions with domain set D and target set C.

(You can write your answer as an algebraic expression.)

$|A| = 3$, $|B| = 4$, $|C| = 4^3 = 64$, *and* $|D| = 3^4 = 81$.

Since the target is of smaller cardinality than the domain, the number of one-to-one functions is zero.

The number of functions from D to C is $|C|^{|D|} = 64^{81}$, but now we want the onto functions. As in the Santa Claus *Problem, set F_c to be the set of functions for which $c \in C$ is not in the image. The set of functions we do* not *want is $\bigcup_{c \in C} |F_c|$, which using inclusion exclusion is*

$$\sum_{k=1}^{64} (-1)^{k-1} \binom{64}{k} (64-k)^{81}$$

So the number of onto functions is

$$64^{81} - \sum_{k=1}^{64} (-1)^{k-1} \binom{64}{k} (64-k)^{81} = \sum_{k=0}^{64} (-1)^{k} \binom{64}{k} (64-k)^{81}$$