

Ma2201/CS2022 Quiz 0100

1. (4 points) Suppose that  $\{p_n\}$  is a set of statements for  $n \in \{1, 2, 3, \ldots\}$ , and suppose that for all n we have  $p_n \Longrightarrow p_{n+1}$ .

Suppose  $p_3 \lor p_5$  is true and  $p_2 \lor p_4$  is true. Circle each of the following which must be true.

b)  $p_1 \vee p_5$ a)  $p_1 \vee p_6$ c)  $p_1 \vee p_4$ d)  $p_1 \vee p_3$ 

We have  $p_2 \vee p_4$  is true. If  $p_2$  is true, then so is  $p_3$  and  $p_4$  by induction. If  $p_2$  is false, then  $p_2 \vee p_4$  implies  $p_4$  is true. So  $p_4$  is true in either case, and by induction so is  $p_5$  and  $p_6$ , so a), b) and c) should be circled.

d) should not be circled since  $p_1$ ,  $p_2$  and  $p_3$  false and all others true is consistent with the assumptions.

2. (6 pts) Prove carefully by induction that  $\left[\sum_{k=1}^{n} k\right] = n(n+1)/2$ .

The statement is  $p_n := [\sum_{k=1}^n k = n(n+1)/2]$ . Proof by induction. Base case:  $p_1 := [\sum_{k=1}^1 k = 1(1+1)/2]$  The right hand side is 1. The left hand side is 1(2)/2 = 1. So  $p_1$  is true.

Induction Step: Assume  $p_n$  is true, that is, assume  $\sum_{k=1}^n k = n(n+1)/2$  is true for a particular value of n.

We want to show  $p_{n+1} := \left[ \sum_{k=1}^{n+1} k = (n+1)(n+2)/2 \right].$ 

$$\sum_{k=1}^{n+1} k = \left(\sum_{k=1}^{n} k\right) + (n+1)$$
  
=  $n(n+1)/2 + (n+1)$  using  $p_n$ , that is, by the induction hypothesis  
=  $(n+1)[n/2+1]$   
=  $(n+1)[(n+2)/2]$   
=  $(n+1)(n+2)/2$ 

as required.

So  $p_{n+1}$  is true and have have shown  $p_n \Longrightarrow p_{n+1}$ .

Conclusion. Since the base case,  $p_1$ , is true and the induction step is true for all n, the statement  $p_n$  is true for all n > 1.