Ma2201/CS2022 | **Discrete Mathematics** | D Term, 2013
Quiz 0011 | Solutions

1. (**6 points**) Suppose p, q and r are statements and suppose $(p \vee \sim r)$ and $(r \vee \sim q)$ are both true.

Circle each of the following which must be true.

a) $p \Longrightarrow r$

b) $q \Longrightarrow r$

c) $(p \wedge r) \vee (p \wedge \sim q) \vee (\sim r \wedge \sim q)$

d) $\sim(r \wedge \sim p)$

e) $(p \wedge r) \vee (r \wedge \sim r) \vee (p \wedge \sim q) \vee (\sim r \wedge \sim q)$

f) $(q \Longrightarrow p) \vee (p \wedge \sim p)$

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a) $(p \vee \sim r)$ is equivalent to $r \Longrightarrow p$, which is the converse of a)

c&e) Applying the distributive law to $(p \vee \sim r) \wedge (r \vee \sim q)$ gives e), and since $(r \wedge \sim r)$ is always false, this is equivalent to c) as well.

b) $q \Longrightarrow r$ and $(r \vee \sim q)$ are equivalent

d) Applying the de Morgan's law to $\sim(r \wedge \sim p)$ gives $(\sim r \vee p)$ which is equivalent to $(p \vee \sim r)$

f) $(p \vee \sim r)$ is equivalent to $r \Longrightarrow q$ and $(r \vee \sim q)$ is equivalent to $q \Longrightarrow r$, so together they imply $q \Longrightarrow p$, so $(q \Longrightarrow p) \vee (p \wedge \sim p)$ is true despite the fact that $p \wedge \sim p$ must be false.

2. (**4 pts**) How many 12 letter strings of lower case letters $L = \{a, b, c, \ldots, z\}$ have either vowels ($V = \{a, e, i, o, u\}$) in the the odd positions, start with three identical letters, or read the same forwards as backwards.

We use inclusion/exclusion.

Let A be the subset of those strings with vowels in the odd positions.

Let B be the subset of those strings which start with three identical letters.

Let C be the subset of those strings which read the same forwards as backwards.

$|A| = 26^6 \cdot 5^6$, *since six of the twelve positions have only six choices.*

$|B| = 26^{10}$, *since the choice of the first letter determines the next two.*

$|C| = 26^6$, *since the first 6 letters may be chosen independently, and the last 6 has no choice.*

$|A \cap B| = 26^5 \cdot 5^5$, *since the vowel in the first position removes the choice the letters in the 2nd and third positions.*

$|A \cap C| = 5^6$, *since it reads forwards and backwards the same, there will be vowels in all positions, and the first six determine the last six.*

$|B \cap C| = 26^4$, *since choosing letters 1, 4, 5 and 6 determine all the rest.*

$|A \cap B \cap C| = 5^4$, *since all the letters now must be vowels, and choosing letters 1, 4, 5 and 6 determine all the rest.*

So altogether we have

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = \\ &= 26^6 \cdot 5^6 + 26^{10} + 26^6 - 26^5 \cdot 5^5 - 5^6 - 26^4 = 5^4 \end{aligned}$$