Ma2201/CS2022 | Discrete Mathematics | D Term, 2013

Quiz 0010 | Solutions

1. (**4 points**) Let A, B, and C be nonempty subsets of $X = \{1, 2, 3, \ldots, 100\}$. Suppose $A = \{1, 2, 3, \ldots, 50\} = \{n \in X \mid n \leq 50\}$ and $B = \{1, 3, 5, 7, 9, \ldots, 99\} = \{n \in X \mid n \text{ is odd}\}$, and $A \cap B \cap C = \emptyset$.

Circle each of the following which must be true.

a) $A \cup B \cup C = X$

b) $(A \cap C) \cup (B \cap C) = \emptyset$

$\boxed{\text{c)}}$ There are $2^{75} - 1$ possibilities for C

d) $|\mathcal{P}(A \cup B)| = \binom{100}{74}$

$A \cap B = \{1, 3, 5, \ldots, 49\}$, with $|A \cap B| = 25$. In order for $A \cap B \cap C$ to be empty, C should be any subset of the remaining 75 elements of X except $\emptyset$, which is specifically excluded.

a) not circled since $C = \{98\}$ is allowed.

b) is not circled since $(A \cap C) \cup (B \cap C) = (A \cup B) \cap C$ and $C = \{99\}$ is allowed.

c) should be circled because the C can be is a any subset of the allowed 75 elements, subtracting 1 for $\emptyset$.

d) No computations are needed for d). The cardinality of a power set is a power of 2.

2. (**3 pts**) Let $Y = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

Circle each of the following which must be true.

$\boxed{\text{a)}}$ $|\mathcal{P}_1(Y)| \in Y$.

b) $\mathcal{P}_0(Y) = \emptyset$.

c) There is an integer k so that $|\mathcal{P}_k(X)| = \mathcal{P}_k(Y)$

$\boxed{\text{d)}}$ $|\mathcal{P}_3(Y)| = \binom{10}{3}$

a) is circled because $\mathcal{P}_1(Y)$ is the set of 1 subsets of Y, which has cardinality 10, and $10 \in Y$.

b) $\mathcal{P}_0(Y)$ is the set of subsets of Y having cardinality 0, so $\mathcal{P}_0(Y) = \{\emptyset\}$, and $|\mathcal{P}_0(Y)| = \{\emptyset\} = 1$. In particular, as we discussed in class, $\{\emptyset\} \neq \emptyset$, for instance because $|\emptyset| = 0$.

c) You need not look for an integer k, the object on the left is always a number, the object on the right is always a set.

d) must be circled. This is the definition.

3. (**3 pts**) Let A, B and C be sets.

Prove directly that $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$.

[Venn diagrams are not a proof.]

Let $x \in (A \cap B) \cup C \subseteq (A \cup C)$. The either $x \in A \cap B$ or $c \in C$.

If $x \in A \cap B$ then $x \in A$ and $x \in B$. Since $x \in A$, we have $x \in A \cup C$. Since $x \in B$, we have $x \in B \cup C$ Since $x \in A \cup C$ and $x \in B \cup C$, we have $x \in (A \cup C) \cap (B \cup C)$, as required in this case.

On the other hand, if $x \in C$, then $x \in A \cup C$ and $x \in B \cup C$, so $x \in (A \cup C) \cap (B \cup C)$ as required in this case as well.

So $(A \cap B) \cup C \subseteq (A \cup C) \cap (B \cup C)$.