Ma2201/CS2022
Quiz 0001

Discrete Mathematics

D Term, 2013
Solutions

1. (**3 points**) You place 13 cards from an ordinary 52 card deck in order on a table. How many ways are there to do this.

a) $\frac{52!}{13!}$ c) $\frac{52!}{13^6}$

$\boxed{\text{b)}}$ $\frac{52!}{39!}$ d) $\frac{52!}{40!}$

Here we have the weaker form independence; each time you place a card, the particular choices for the next card changes, but the number of choices is the same. So the multiplicative principle holds. Since you are taking 13 cards and each card placed reduces choice for the next card we have:

$$52 \cdot (52-1) \cdot (52-2) \cdots (52-12) = 52 \cdot 51 \cdot 50 \cdots 40.$$

As shown in class, multiplying upstairs and downstairs by 39! gives that b is correct.

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2. (**3 pts**) Before tying Albertson's magic trick, the MC lost the third card and quickly made a new one using a selection of numbers from the first two cards. So the third answer cannot be yes, if the first two answers are no. If this is the only dependence introduced, how many different yes-no sequences can there be? Justify your answer. (Do not simply list them all.)

Under the assumption that no-no-yes for the first three cards is the only dependence introduced, we can use the multiplicative principle to see which answers are excluded over five questions. Starting with no-no-yes There are two answers for the fourth card that cannot occur, and for each of these there are two answers on the 5th card, so $2^2 = 4$.

So there are So of the 32 yes/no sequences 4 are excluded and there are $\boxed{32 - 4 = 28}$ *yes no answers. [So if he does that there is no way to distinguish the 31 numbers.]*

Note: The problem asks for sequences if no-no-no is the only dependence, but the MC probably created many more, for instance yes-yes-yes is also dependence.

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3. (**4 pts**) How many numbers have at most 6 digits and have at least one 7 or one 5? You may express your answer as an algebraic expression.

By the multiplicative principle there are 10^6 *numbers with at most 6 digits.*

We are excluding the numbers without 7's or 5's, of which there are 8^6*, again by the multiplicative principle.*

So there are $10^6 - 8^6$.
