Ma2201/CS2022

Discrete Mathematics

D Term, 2012

Quiz 0101

PRINT NAME: ____________________

$\mathcal{SIGN}$: ____________________

1. (3 pts) How many ways are there to distribute 101,101 (indistinguishable) pennies among 101 (distinguishable) children?

Solution: Make a row of markers which can stand for either a penny, or a partition between pennies. We need 101,101 markers for the pennies, and 101-1 = 100 markers to partition the pennies for distribution among the 101 children. So there is one distribution for each choice of 100 partitions among the 101,201 markers, so $\binom{101{,}201}{100}$ ways to distribute.

2. (3 pts) How many ways are there to distribute 101,101 (indistinguishable) pennies among 101 (distinguishable) children such that each child gets at least 11 pennies?

Solution: Setting aside $101 \cdot 11 = 1,111$ for the minimum distribution for each child, we have to distribute the remaining 99,990 pennies among the 101 children, using the method above, so $\binom{100{,}090}{100}$ distributions.

3. (4 pts) Compute

$$\sum_{k=0}^{101}(-1)^k\binom{101}{k}2^k = 1 - 101\cdot 2 + \binom{101}{2}2^2 - \binom{101}{3}2^3 + \binom{101}{4}2^4 - \cdots - \binom{101}{101}2^{101}$$

Solution: Gathering terms we have

$$\sum_{k=0}^{101}(-1)^k\binom{101}{k}2^k = \sum_{k=0}^{101}\binom{101}{k}(-2)^k\cdot 1^{101-k} = (-2+1)^{101} = (-1)^{101} = -1$$

by the Binomial Theorem.

[Buy the *Binomial Theorem*! Buy now and get 10% off the *Sieve of Eratosthenes* and a coupon on your next purchase of any Lemma by the Bernoulli Brothers.]