Ma2201/CS2022
Quiz 0100

Discrete Mathematics

D Term, 2012

PRINT NAME: __________________
SIGN: __________________

1. (5 pts) Two words are called *isoterminyms* if they have the same first letter and the same last letter. So, for example, **o**ne-hundre**d** and **o**ne-million-seventeen-thousan**d** are isoterminyms with terminal letters **o** and **d**, and likewise **s**i**x** and **s**eventy-si**x** are isoterminyms with terminal letters **s** and **x**.

Prove that any subset of fifty numbers taken from the set of numbers $\{1, \ldots, 100\}$ contains at least one pair of numbers which are, when spelled out in English, isoterminyms.

Solution: The set of first letters $\{e, f, n, o, s, t\}$ has 6 elements. The set of last letters $\{d, e, n, o, r, t, x, y\}$ has 8 elements. So, by the multiplicative principle, the set of pairs of first and last letters can have at most $6 \cdot 8 = 48$ elements. So if the domain is any subset of 50 numbers, and the function sends each number to the pair of first and last letters, the target is of size $48 < 50$, and the pigeonhole principle implies that the function is not one-to-one, hence there are two numbers with the same first and last letters.

Note that all the numbers beyond 100 are spelled starting with a number between one and 100 in the front, and end either with a number from one to 99, or with a d for hundred or thousand, or an n for million, billion, trillion, quadrillion, etc. So there are no new starting or ending letters, and so any *set of 49 positive integers which have English names contains at least two isoterminymns.*

2nd Solution A much more involved proof could avoid the multiplicative principle by computing precisely all the pairs which actually do occur in the words for the numbers from 1 to 100. This solution would require a proof of why the set is correct.

2. (5 pts) Compute, using inclusion/exclusion, how many numbers from 1 to 1000 are either even, or are between 100 and 200, or end in a 5 or a 6.

Solution: Let $A = \{n \mid 1 \leq n \leq 1000, n = 2k\}$, $B = \{n \mid 100 \leq n \leq 200\}$, and $C = \{n \mid 1 \leq n \leq 1000, n \text{ ends in a 5 or 6}\}$.

$$\begin{aligned} |A \cup B \cup C| &= |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| \\ &= 500 + 101 + 200 - 51 - 100 - 20 + 10 \\ &= 811 - 171 = 640. \end{aligned}$$