Ma2201/CS2022 **Discrete Mathematics** D Term, 2012

Quiz 0011

PRINT NAME: ________________

$\mathcal{SIGN}$: ________________

1. (6 pts) Prove by induction that $\displaystyle\sum_{k=0}^{n}(2k+1) = (n+1)^2$ for all $n \geq 0$.

Proof by induction on n: THE BASE CASE: $n = 0$. The statement is $\displaystyle\sum_{k=0}^{0}(2k+1) = (0+1)^2$.

Since $\displaystyle\sum_{k=0}^{0}(2k+1) = (2\cdot 0+1) = 1$ and $(0+1)^2 = 1^2 = 1$ the base case is true.

INDUCTIVE STEP: We assume the *induction hypothesis* $\displaystyle\sum_{k=0}^{n}(2k+1) = (n+1)^2$ and must show that $\displaystyle\sum_{k=0}^{n+1}(2k+1) = ((n+1)+1)^2$.

$$
\begin{aligned}
\sum_{k=0}^{n+1}(2k+1) &= \left[\sum_{k=0}^{n}(2k+1)\right] + (2(n+1)+1) \\
&= (n+1)^2 + (2(n+1)+1) \qquad \textit{by the induction hypothesis} \\
&= (n^2+2n+1) + (2n+3) \\
&= n^2+4n+4 \\
&= (n+2) \qquad \textit{as required}
\end{aligned}
$$

So the result is true by induction.

2. (4 pts) Let $\{\mathcal{P}(n)\}$ be a sequence of statements depending only on the non-negative integer n.

Suppose $\mathcal{P}(n)$ implies $\mathcal{P}(n+3)$ for all $n \geq 0$, that is

$$\mathcal{P}(n) \Longrightarrow \mathcal{P}(n+3)$$

Suppose further that $\mathcal{P}(0)$ is true and $\mathcal{P}(100)$ is false.

Label each of the following true or false.

$\underline{\;\;T\;\;}$ $\mathcal{P}(99)$ must be true. *Since $\mathcal{P}(0)$ is true and inductively*

$$\mathcal{P}(0) \Rightarrow \mathcal{P}(3) \Rightarrow \mathcal{P}(6) \Rightarrow \mathcal{P}(9) \Rightarrow \mathcal{P}(12) \Rightarrow \mathcal{P}(15) \Rightarrow \cdots \Rightarrow \mathcal{P}(99)$$

$\underline{\;\;T\;\;}$ $\mathcal{P}(999)$ must be true. *For the same reason.*

$\underline{\;\;F\;\;}$ $\mathcal{P}(101)$ must be false. *Might be true or false. If any of $\mathcal{P}(2)$, $\mathcal{P}(5)$, $\mathcal{P}(6)$, $\cdots$ are true $\mathcal{P}101$ would have to be true.*

$\underline{\;\;T\;\;}$ $\mathcal{P}(1)$ must be false. It must be false. Otherwise, if $\mathcal{P}(1)$ is true, then

$$\mathcal{P}(1) \Rightarrow \mathcal{P}(4) \Rightarrow \mathcal{P}(7) \Rightarrow \mathcal{P}(10) \Rightarrow \cdots \Rightarrow \mathcal{P}(100)$$

and so $\mathcal{P}(100)$ is true, contrary to the given assumption.