Ma2201/CS2022 **Discrete Mathematics** D Term, 2012
Quiz 0010

Print Name: ____________________

Sign: ____________________

1. (**4 pts**) Let $A = \{1, 3, 5, 7\}$, $B = \{2, 4, 6, 8\}$ and $C = \{1, 2, 3, 4\}$. Give the cardinality of each of the following.

__8__ $A \cup B \cup C$ __0__ $A \cap B \cap C$

_256_ The set of all subsets of $A \cup B \cup C$ __1__ The set of all subsets of $A \cap B \cap C$

$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $|A \cup B \cup C| = 8$.
$A \cap B \cap C = \emptyset = \{\ \}$, $|\emptyset| = 0$.
The set of all subsets of $|A \cup B \cup C|$ has cardinality $2^{|A \cup B \cup C|} = 2^8 = 256$
The set of all subsets of $|A \cap B \cap C|$ has cardinality $2^{|A \cap B \cap C|} = 2^0 = 1$
(In other words $|\{\emptyset\}| = 1$)

2. (**4 pts**) Let X, Y and Z be sets. Prove directly that $X \cup (Y \cap Z) \subseteq (X \cup Y) \cap (X \cup Z)$.

Solution:
Let $a \in X \cup (Y \cap Z)$. Then $a \in X$ or $a \in Y \cap Z$.
If $a \in X$, then $a \in X \cup Y$ and $a \in X \cup Z$, so $a \in (X \cup Y) \cap (X \cup Z)$.

On the other hand, if $a \in Y \cap Z$, then $a \in Y$ and $a \in Z$. Since $a \in Y$ then $a \in X \cup Y$. Since $a \in Z$, $a \in X \cup Z$. So $x \in (X \cup Y) \cap (X \cup Z)$.

So, in either case $a \in (X \cup Y) \cap (X \cup Z)$ as required.

Note: A correct "Venn diagram proof" would require an explanation of how and why your shaded diagrams are arrived at, and what they indicate.

3. (**2 pts**) Let $D = \{1, 2\}$, E be the set of all subsets of D, and F be the set of all subsets of E. What is $E \cap F$?

Solution: The elements of E are sets of integers: $E = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.
The elements of F are sets of sets of integers $F = \{\emptyset, \{\emptyset\}, \{\{1\}\}, \{\{2\}\}, \{\{1, 2\}\},$
$\{\emptyset, \{1\}\}, \{\emptyset, \{2\}\}, \{\emptyset, \{1, 2\}\}, \{\{1\}, \{2\}\}, \{\{1\}, \{1, 2\}\}, \{\{2\}, \{1, 2\}\},$
$\{\emptyset, \{1\}, \{2\}\}, \{\emptyset, \{1\}, \{1, 2\}\}, \{\emptyset, \{2\}, \{1, 2\}\}, \{\{1\}, \{2\}, \{1, 2\}\}, \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}\}$.
The only element they have in common is the empty set, so $E \cap F = \{\emptyset\}$.
(Note: This is not the same as $E \cap F = \emptyset$, which is false.)