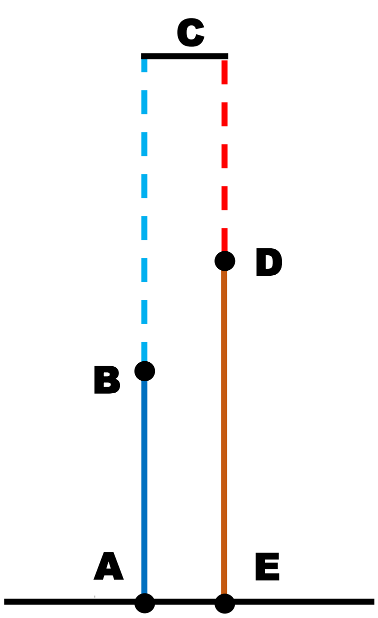
Uber Rocket Problem (Calculus)

Henry Liu Section S October 6, 2021

Problem Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Diagram



Givens

$a_{AB}[t] = -0.5t^2 + 18 \text{ m/s}^2$	$v_A = 0 \text{ m/s}$
$t_B = 7.0 \text{ sec}$	$y_A = 0 \text{ m}$
$\Delta y_{CD} = -131 \text{ m}$	$t_A = 0 \text{ sec}$
$v_{DE}[t] = -17\left(1 - e^{-\frac{1}{8}}\right) \text{ m/s}$	$v_c = 0 \text{ m/s}$
$a_{BD} = -9.8 \text{ m/s}^2$	$y_B = 0 \text{ m}$
$a_{BC} = -9.8 \text{ m/s}^2$	$y_E = 0 \text{ m}$

Strategy

To solve this problem, I will split the motion of the rocket into 3 stages: Stage AB, Stage BD, and Stage DE.

For stage AB, the indefinite integral of $a_{AB}[t]$ can be taken to find $v_{AB}[t] = \Delta v_{AB} + C$, where v_A can be substituted for the constant C. Using the equation for $v_{AB}[t]$, y_B can be found by solving $v_{AB}[t]$ when $t = t_B$. In addition, by taking the definite integral of $a_{AB}[t]$ from $t = 0$ to t_B, v_B can be found.

Next, for stage BD, the rocket is in projectile motion with constant acceleration. Knowing v_c, v_B , and a_{BD}, Δy_{BC} can be found and can then be added to y_B to find y_C. Adding Δy_{CD} to y_C will then find y_D. With these values for a_{BD}, v_B, y_B, and y_D, t_{BD} can be found.

Then, for stage DE, the equation for v_{DE} is given. Taking the integral of v_{DE} and solving for C will get us the equation for y_{DE}. Solving the equation when $y_{DE} = y_E$ for t will find us t_{DE}.

Finally, taking the sum of t_B, t_{BD}, and t_{DE} will find us t_{Final}, or the total time the rocket is in the air.

Stage AB

First, the integral of $a_{AB}[t]$ is taken. Substituting v_A for the constant C will get us the equation for $v_{AB}[t]$:

$$a_{AB}[t] = -0.5t^2 + 18$$
$$v_{AB}[t] = \int a_{AB}[t]dt$$
$$v_{AB}[t] = \int (-0.5t^2 + 18)dt \ + C$$
$$v_{AB}[t] = -\frac{1}{6}t^3 + 18t + v_A$$
$$v_{AB}[t] = -\frac{1}{6}t^3 + 18t$$

Substituting t_B in for t will then find us v_B:

$$v_{AB}[t] = -\frac{1}{6}t^3 + 18t$$
$$v_{AB}[t_B] = -\frac{1}{6}t_B^3 + 18t_B$$
$$v_{AB}[7] = -\frac{1}{6}(7)^2 + 18(7)$$
$$\underline{v_B = 68.833 \text{ m/s}}$$

Finally, taking the definite integral of $v_{AB}[t]$ from t=0 to 7 will find us y_B:

$$y_B = \int_{t=t_A}^{t_B} v_{AB}[t]dt$$
$$y_B = \int_{t=0}^{7} (-\frac{1}{6}t^3 + 18)\,dt$$
$$\underline{y_B = 340.958 \text{ m}}$$

Stage BD

First, Δy_{BC} can be found knowing a_{BC}, v_B, and v_c:

$$v_C^2 = v_B^2 + 2a_{BC}\Delta y_{BC}$$
$$0 = (68.833)^2 + 2(-9.8)\,\Delta y_{BC}$$
$$19.6\Delta y_{BC} = 4738.03$$
$$\underline{\Delta y_{BC} = 241.734 \text{ m}}$$

Adding Δy_{BC} to y_B will find us y_C:

$$y_C = y_B + \Delta y_{BC}$$
$$y_C = 340.958 + 241.734$$
$$\underline{y_C = 582.692 \text{ m}}$$

Next, adding Δy_{CD} to y_C will get us y_D:

$$y_D = y_C + \Delta y_{CD}$$
$$y_D = 582.692 + (-131)$$
$$\underline{y_D = 451.692 \text{ m}}$$

Knowing a_{BD}, v_B, y_B, and y_D, t_{BD} can be found:

$$y_D = \frac{1}{2}a_{BD}t_{BD}^2 + v_B t_{BD} + y_B$$
$$451.692 = \frac{1}{2}(-9.8)t_{BD}^2 + 68.833t_{BD} + 340.958$$
$$0 = -4.9t_{BD}^2 + 68.833t_{BD} - 110.734, \text{SOLVER}$$
$$\underline{t_{BD} = 12.1943 \text{ sec}}$$

Stage DE

Find y_{DE} by taking the indefinite integral of v_{DE}:

$$y_{DE}[t] = \int v_{DE}dt$$
$$y_{DE}[t] = \int [-17(1 - e^{-\frac{t}{8}})]\,dt$$
$$y_{DE}[t] = -17t - 136e^{-\frac{t}{8}} + C$$

Next, solve for C knowing that $y_{DE}[0] = y_D$ to find the equation for $y_{DE}[t]$:

$$y_{DE}[0] = y_D$$
$$y_{DE}[t] = -17t - 136e^{-\frac{t}{8}} + C$$
$$y_{DE}[0] = -17(0) - 136e^{\frac{0}{8}} + C = y_D$$
$$451.692 = -136 + C$$
$$\underline{C = 582.692}$$
$$y_{DE}[t] = -17t - 136e^{-\frac{t}{8}} + C$$
$$y_{DE}[t] = -17t - 136e^{-\frac{t}{8}} + 582.692$$

Knowing the equation for $y_{DE}[t]$ as well as y_E, t_{DE} can be found:

$$y_{DE}[t] = -17t - 136e^{-\frac{t}{8}} + 582.692$$
$$y_E = y_{DE}[t_{DE}] = -17t_{DE} - 136e^{-\frac{t_{DE}}{8}} + 582.692$$
$$0 = -17t_{DE} - 136e^{-\frac{t_{DE}}{8}} + 582.692, \text{SOLVER}$$
$$\sout{t_{DE} = -14.5125 \text{ sec}} \text{ or } \underline{t_{DE} = 34.4624 \text{ sec}}$$

Final Sum

Finding the sum of t_B, t_{BD}, and t_{DE} will get us the value for t_{Final}, or the total time the rocket spent in motion, which is the final answer:

$$t_{Final} = t_B + t_{BD} + t_{DE}$$
$$t_{Final} = 7.0 + 12.1943 + 34.4624$$
$$\boxed{t_{Final} = 53.66 \text{ sec}}$$

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Tables and Graphs

t	y	v	a
(s)	(m)	(m/s)	(m/s^2)
0.00	0.00	0.00	18.00
2.00	35.33	34.67	16.00
4.00	133.33	61.33	10.00
6.00	270.00	72.00	0.00
7.00	340.96	68.83	-6.50
7.00	340.96	68.83	-9.80
8.00	404.89	59.03	-9.80
10.00	503.36	39.43	-9.80
12.00	562.63	19.83	-9.80
14.00	582.69	0.23	-9.80
16.00	563.56	-19.37	-9.80
18.00	505.23	-38.97	-9.80
19.19	451.70	-50.67	-9.80
19.19	451.70	0.00	-2.13
20.00	451.03	-1.63	-1.92
22.00	444.23	-5.03	-1.50
24.00	431.42	-7.68	-1.17
26.00	413.91	-9.74	-0.91
28.00	392.76	-11.35	-0.71
30.00	368.77	-12.60	-0.55
32.00	342.56	-13.57	-0.43
34.00	314.63	-14.33	-0.33
36.00	285.36	-14.92	-0.26
38.00	255.04	-15.38	-0.20
40.00	223.91	-15.74	-0.16
42.00	192.14	-16.02	-0.12
44.00	159.88	-16.23	-0.10
46.00	127.23	-16.40	-0.07
48.00	94.29	-16.54	-0.06
50.00	61.11	-16.64	-0.05
52.00	27.75	-16.72	-0.04
53.66	0.00	-16.77	-0.03