Uber Pulley Problem (Calculus)

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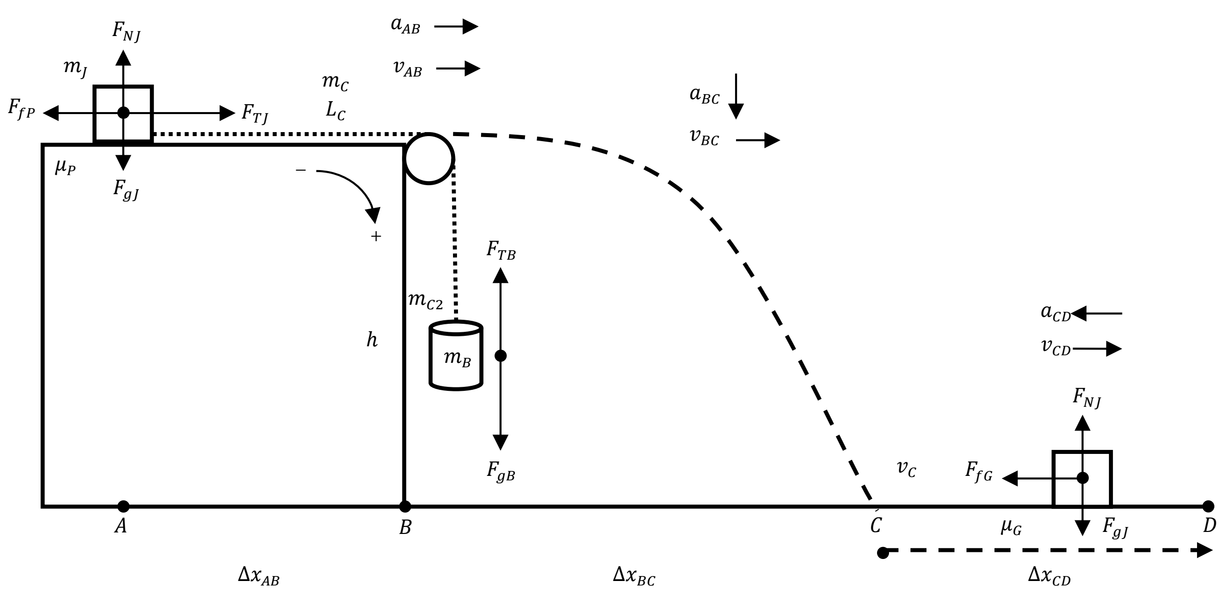
Problem Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper, he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his (net) speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain



Givens and Assumptions

$$m_J = 64\ kg \qquad h = 19\ m$$

$$m_B = 154\ kg \qquad \mu_P = 0.24$$

$$m_C = 44\ kg \qquad \Delta x_{BD} = 77\ m$$

$$L_C = \Delta x_{AB} = 9\ m \qquad F_{TJ} = F_{TB}$$

Strategy

The problem can be solved by separating the problem into 3 stages: the pulley and chain system (stage AB), the project motion stage (stage BC), and the stage where the jumper is sliding on the floor (stage CD).

Stage AB

Begin by using $\sum F_{yAB}$ to find F_N:

$$\sum F_{yAB}: F_{NJ} - F_g = ma$$

$$F_{NJ} = (m_J g) + ma$$

$$F_{NJ} = (64)(9.8) + 0$$

$$\underline{F_{NJ} = 627.2\ N}$$

Then, solve for F_{fP} using F_N:

$$F_{fP} = \mu_P F_{NJ}$$

$$F_{fP} = (0.24)(627.2)$$

$$\underline{F_{fP} = 150.5\ N}$$

The mass of the chain over the table, m_{C2}, can be found using L_C and m_C, where x is the amount that the chain has moved:

$$m_{C2} = \frac{m_C}{L_C}(x)$$

$$\underline{m_{C2} = \frac{44x}{9}}$$

Then, solve for $a[x]$ using the sum of the forces in the system as well as the system axis:

$$\sum F_S\colon\ F_{gB} - F_{T2} + F_{T1} - F_{fP} = m_s a$$

$$g(m_B + m_{C2}) - F_{fP} = \left(m_J + m_B + m_C\right)a$$

$$(9.8)\left(154 + \frac{44x}{9}\right) - 150.5 = (64 + 154 + 44)a$$

$$\underline{a[x] = 0.1829x + 5.1858}$$

Finally, solve for v_B using $a[x]$:

$$\int_{x_0}^{x_f} a[x]dx = \int_{v_0}^{v_f} v dv$$

$$\int_0^9 (0.1829x + 5.1858)dx = \int_0^{v_B} (v)dv$$

$$54.0797 = \frac{1}{2}v_B^2$$

$$\underline{\overrightarrow{v_B} = 10.4\ m/s}$$

Stage BC

Start by solving for t_{BC} using g for a_y:

$$h = \frac{1}{2}a_y t_{BC}^2 + v_{By}t$$

$$-19 = \frac{1}{2}(-9.8)t_{BC}^2 + 0$$

$$\underline{t_{BC} = 1.969\ s}$$

Then, solve for Δx_{BC} using t_{BC}:

$$\Delta x_{BC} = \frac{1}{2}a_x t_{BC}^2 + v_B t$$

$$\Delta x_{BC} = 0 + (10.4)(1.969)$$

$$\underline{\Delta x_{BC} = 20.479\ m}$$

Finally, solve for v_{yC}, which can then be used with v_{xC} to find v_C, of which 75% is converted to v_{CD}:

$$v_{yC}^2 = v_{yB}^2 + 2ah$$

$$v_{yC} = \sqrt{2(-9.8)(-19)}$$

$$\underline{v_{yC} = 19.298\ m/s}$$

$$v_{CD} = (0.75)\sqrt{v_{xC}^2 + v_{yC}^2}$$

$$v_{CD} = (0.75)\sqrt{(10.4)^2 + (19.298)^2}$$

$$\underline{\overrightarrow{v_{CD}} = 16.44\ m/s}$$

Stage CD and Final Solution

Start by finding Δx_{CD} and a_{CD} using known information:

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$

$$\Delta x_{CD} = 77 - 20.48$$

$$\underline{\Delta x_{CD} = 56.52\ m}$$

$$V_D^2 = V_{CD}^2 + 2a_{CD}\Delta x_{CD}$$

$$0 = (16.44)^2 + 2a_{CD}(56.52)$$

$$\underline{a_{CD} = -2.39\ m/s^2}$$

Use the sum of forces F_{Gy} during stage CD to find F_{NJ}:

$$\sum F_{Gy} : F_N - F_g = ma$$

$$F_N = (64)(9.8) + 0$$

$$\underline{F_N = 627.2\ N}$$

Finally, solve for μ_G by using $\sum F_{xG}$:

$$\sum F_{xG} : -F_{fG} = m_J a_{CD}$$

$$-\mu_G F_N = m_J a_{CD}$$

$$-\mu_G = \frac{(64)(-2.39)}{627.2}$$

$$\boxed{\mu_G = 0.2440}$$