Problem statement:

A puck that has a mass of 72 grams is placed at the top of a 2.9-meter ramp that is inclined at an angle 38 degrees above the horizontal. The coefficient of friction between the surface of the ramp and the puck is 0.17. The bottom of the ramp is at the top of a 1.6-meter-tall counter.

A. Our task is to find how far away the puck lands from the base of the counter.

B. Our other task is to find the best angle to maximize how far the puck will land. This can be achieved by either using a spreadsheet or a computer program.

Process:

We first tried to solve the problem for the distance at an angle of 38 degrees. To begin, we found the final velocity when the puck reached the end of the ramp. We did this by adding all forces on the puck parallel to the ramp, which was the sine of gravity and the frictional force. We then realized that the next part of the problem had to be treated like a projectile motion problem. So, we found the horizontal and vertical components of our velocity and used the no v equation to find the time that the puck spent getting from the end of the ramp to the ground. Afterward, we found the distance by multiplying the horizontal component of the velocity by the time found previously, which gave us an answer of 1.36 meters.

Finding the angle that delivers the maximum distance was much harder for us, however. We realized that trying to do the calculation manually would take too much time.

Therefore, we decided to make a Python program that would run through all possible angles by itself. To do this, we created formulas for all the values that would change because of the angle. These values were velocity, time, and distance. We went to the whiteboard to write out all the variables in terms of all the other values we knew, such as gravity, the height of the drop, the length of the ramp, and the angle. To find the maximum distance, we needed to find the distance that every angle from 0 to 90 degrees would output. We created an iterative loop that would apply the formulas below on every angle in the range, and the angle that caused the maximum distance was printed in the console at the end.

Solution:

$$v = \sqrt{(g\sin(\Theta) - g \cdot \mu\cos(\Theta)) \cdot 2 \cdot L}$$

$$t = \frac{-v\sin(-\Theta) \pm \sqrt{(v\sin(-\Theta))^2 + 2g \cdot H}}{-g}$$

$$d = v\cos(\Theta) \cdot t$$

First, we started by calculating the acceleration of the puck as it slides down the ramp.

Using $F = ma = mg\sin(\theta) - \mu\, mg\cos(\theta)$

We could simplify it as

$$a = g\sin(\theta) - \mu\, g\cos(\theta)$$

$$a = (9.8)\sin(38) - (0.17)(9.8)\cos(38)$$

$$a = 4.72065654\ \frac{m}{s^2}$$

Using the acceleration, we can use the no t equation to find the final velocity after the puck covers the distance of the ramp.

$$v = \sqrt{(g\sin(\Theta) - g \cdot \mu\cos(\Theta)) \cdot 2 \cdot L}$$

$$v^2 = v_o + 2a\Delta x$$

$$v^2 = 0 + 2(4.72)(2.9)$$

$$v^2 = 27.376$$

$$v = 5.23\,\frac{m}{s}$$

Now that we know the velocity, the puck leaves the ramp and becomes a projectile, we have to solve how far it will go by breaking the velocity into its horizontal and vertical components.

First, we need to find how long the puck stays in the air.

$$y = y_o + v_o t - \frac{1}{2}gt^2$$

$$y = 1.6 + 5.23\sin(-38)\,t - \frac{1}{2}gt^2$$

The reason we have a sin (-38) in the equation is to represent the vertical velocity of the puck, which is 38 degrees below the horizontal.

$$t = \frac{-(5.23\ \cos(-38)) \pm \sqrt{(5.23\ \cos(-38))^2 - 4\left(-\frac{1}{2}\cdot 9.8\right)(1.6)}}{2\left(-\frac{1}{2}\cdot 9.8\right)}$$

After applying the quadratic formula, we solve for t as:

$t = 0.33$ Seconds

Now that we know the time it stays in the air, we can use it to find the horizontal distance covered within that time.

$$x = vt$$

$$x = 5.23\cos(-38)\,(0.33)$$

$$x = 1.36\,m$$

For part b, we decided to apply the same equations discussed above, but instead, change the angle. We incremented the angle by 0.01 degrees and compared the result using a for loop in python

```
import math


temp = 0
highestAngle = 0
for i in range(0, 9100): #cycles through every 0.01 degree
    angle = i/100
    time = 0
    acceleration=9.8*math.sin(math.radians(angle)) -
0.17*math.cos(math.radians(angle))*9.8 #finds acceleration
    try:
        velocity = math.sqrt(2*2.9*acceleration) #finds velocity
        print(f"Velocity: {velocity} {angle}")
    except ValueError:
        continue
    time1 = (-(velocity*math.sin(math.radians(-angle))) -
math.sqrt(math.pow((velocity*math.sin(math.radians(-angle))), 2)+2*9.8*1.6))/-9.8
    time2 = (-(velocity*math.sin(math.radians(-angle))) +
math.sqrt(math.pow((velocity*math.sin(math.radians(-angle))), 2)+2*9.8*1.6))/-9.8
#calculates time

    if time1>time2: #takes the time, picks the larger answer
        time = time1
    else:
        time=time2
    print(f"Time: {time}")
    xDistance = velocity*math.cos(math.radians(-angle))*time
    print(f"Distance: {xDistance}")
    if(xDistance>temp): #compares the distance to the largest distance, if it's
over the temporary value then the value updates
        temp = xDistance
        highestAngle = i/100

print(temp)
print(highestAngle)
```

Solution: 26.65 degrees, 1.518 meters

All the comments in the code start with a # and then proceed with a comment

Note: all degrees that are below 9.65 wouldn't work as the puck would not move since the friction force would stop it from moving.

We know that our work is correct because we plugged in the same values we were given and received the same answer that we got when we did it manually. Additionally, we also checked in with other groups and all received the same distance.

Extension:

1. Calculate the distance the puck landed away from the counter X_{BC} if the puck had initial velocity up the ramp of 8.0 m/s. The puck has the same mass, but the length of the ramp is infinitely long meaning no need to worry about it flying off the ramp.
2. Calculate the static coefficient of friction between the puck and the ramp if the puck weighed the same and had an acceleration of 2.6m/s^2