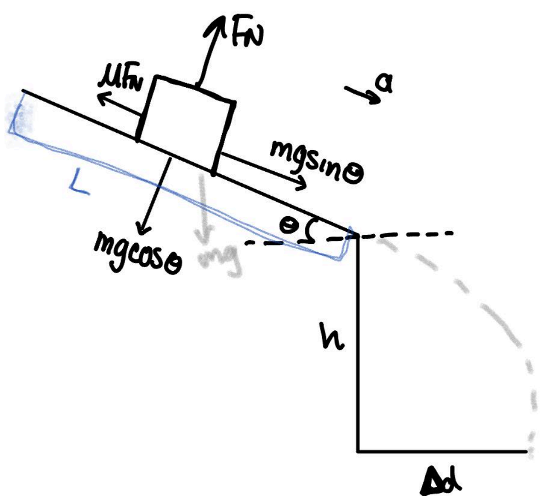
Physics POW #1 - Ramp-Projectile Optimization

Problem Statement

In this POW, there is a 72 g puck of mass sliding down a 2.9 m ramp that is at an angle of 38°, relative to the horizontal. There are numerous forces acting on the puck, including gravity (9.8 m/s^2), the normal force, and friction (coefficient of friction = .17). After determining the acceleration of the puck and how far it would travel at the current angle, we were tasked with trying to determine what the optimal angle for the ramp would be. That is, the angle of the ramp in which the puck travels the farthest after sliding off the edge of the ramp.

Process

Part A:



When attempting to solve the problem, we first wrote down all of our givens in a list and drew a free body diagram for the problem. The puck has a mass of 72 grams which is also equivalent to 0.072 kg and the distance of the ramp is 2.9 meters. Since there is an inclined plane with an angle of 38 degrees, gravity has two components: mgcosθ (perpendicular to plane) and mgsinθ (down along the plane). According to the free body diagram shown below, the normal force is equal to mgcosθ due to acceleration being 0 in the vertical direction. Next, we look at the horizontal components and use Fnet = ma to find acceleration. There is friction (also known as **μ***Normal Force) working against the puck so the arrow would go in the opposite direction of acceleration. The formula for Fnet in the horizontal direction is going to be: mgsinθ - **μ**FN = ma. We can substitute the expression mgcosθ for FN and plug in all corresponding numerical values to solve for acceleration.

Next, we used the acceleration of the puck on the inclined plane to find the final velocity the puck will have before it is projected off the plane onto the ground. Since the time that the puck was on the inclined plane is not given in the problem, we utilized the no t kinematic equation to solve for final velocity: $V^2 = Vo^2 + 2a\Delta d$. The initial velocity is 0 as the object started at rest and the change in distance is 2.9 meters because it is the length of the ramp. Once all values are plugged in, the final velocity is solved for. It is important to keep in mind that the final velocity solved for the ramp is equal to the initial velocity for the projectile once the puck flies off the counter.

Moreover, we find the distance the puck lands away from the counter by solving for time that the puck is in the air for (vertical component) and then, solving for distance (horizontal component). We found time by applying the no V formula: $X = Xo + Vot + 1/2at^2$. Here, the acceleration is -9.8 because there is gravity acting on the puck and pulling it down. The initial velocity would be the vertical component (Vsinθ) of the velocity found before because velocity is a vector. Additionally, the change in x is Xo - X = 1.6 which is given in the problem. When we plugged the equation into the graphing calculator, the time it took for the puck to reach the ground from the ramp was found.

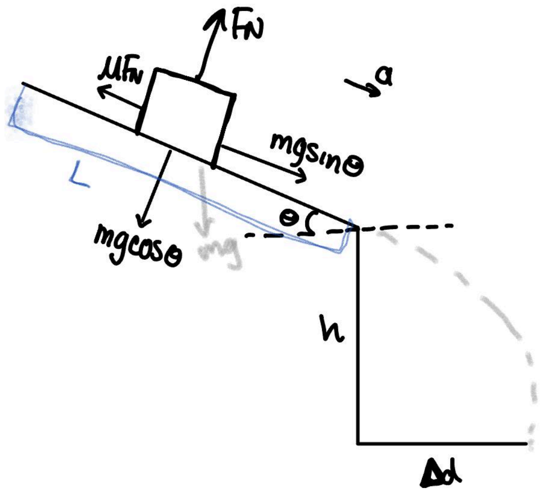
Lastly, we found the distance by using the equation $\Delta x = vt$. The velocity is the horizontal vector component (Vcosθ) and the time was already found from the quadratic equation in the step above. Thus, this is how we found the distance (X_{BC}) that the puck lands away from the base of the counter.

Part B:

The main strategy for finding the optimal angle to put the puck the farthest distance from the ramp was to write a series of equations based on constant information (numbers that remain the same through the changing angle) and use the angle as the x-value in a graph to find the distance as the y-value. Entering values that stay the same while the angle changes gives a more consistent equation, while still allowing the angle to be varied. Deriving the equation for acceleration from the dynamics equations used to find the acceleration of our given angle, and using that equation in both the velocity and time equations, allowed us to write all equations into a graphing software (Desmos in this case) and use the outputted variables to graph the final equation with x representing theta. We could then find the maximum y-value of the graph to find the farthest distance possible, and its corresponding angle.

Solution

Part A:



Givens:

L = 2.9m

Mass = 7.2 grams => 7.2/1000= 0.072kg (This is because 1 kg is equal to 1000 grams)

θ=38 degrees

Coefficient of friction μ = 0.17

h=1.6m

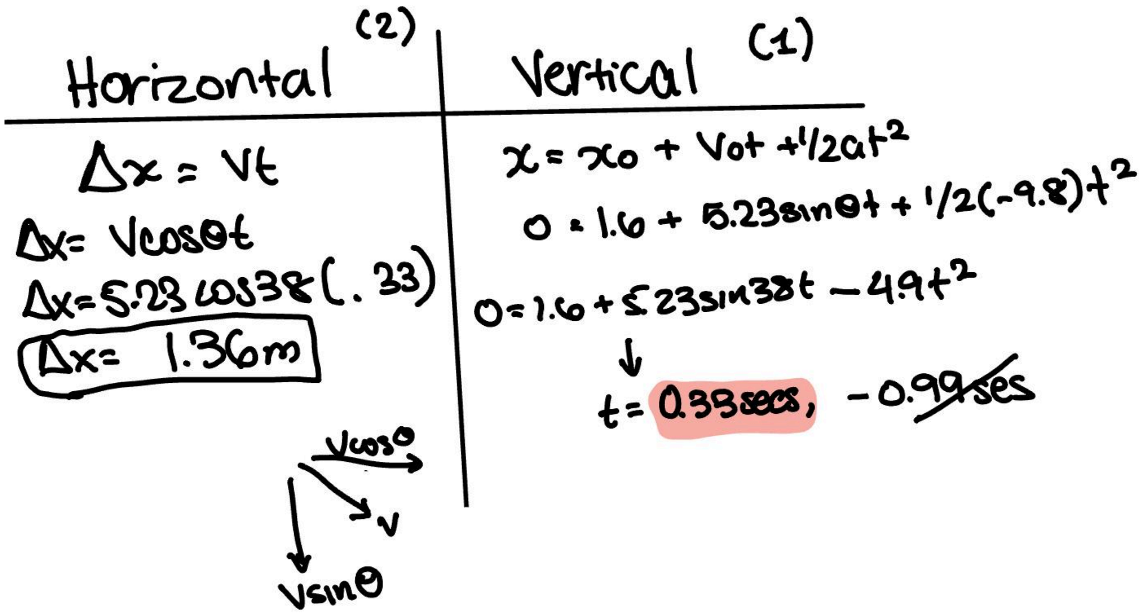
1. Vertical components of Puck: F_N - mgcosθ = ma.
2. Acceleration is 0 because there is no movement in the vertical direction, so the normal force is equal to mgcosθ. Equation: F_N=mgcosθ
3. Next, we looked at the horizontal components and noticed that friction is working against mgsinθ. Force of friction = μF_N. Equation: mgsinθ - μF_N = ma
4. To solve for a, we first substituted the equation from step 2 for F_N.. Simplified form: mgsinθ - μmgcosθ = ma.
5. Plugging in the numerical values, we got: $(0.072 * 9.8 * sin\theta) - (0.17 * 0.072 * 9.8 * cos\theta) = 0.072a$
6. We found that the acceleration is 4.72 m/s^2.
7. Next, we used the no-t formula to solve for V final since Vo would be 0 because the puck starts at rest: V^2 = Vo^2 + 2aΔd. Δd is the length of the ramp so it would be 2.9.
8. When we plugged in the numbers, we got:

V^2 = 0^2 + 2(4.72)(2.9)

V = 5.23 m/s

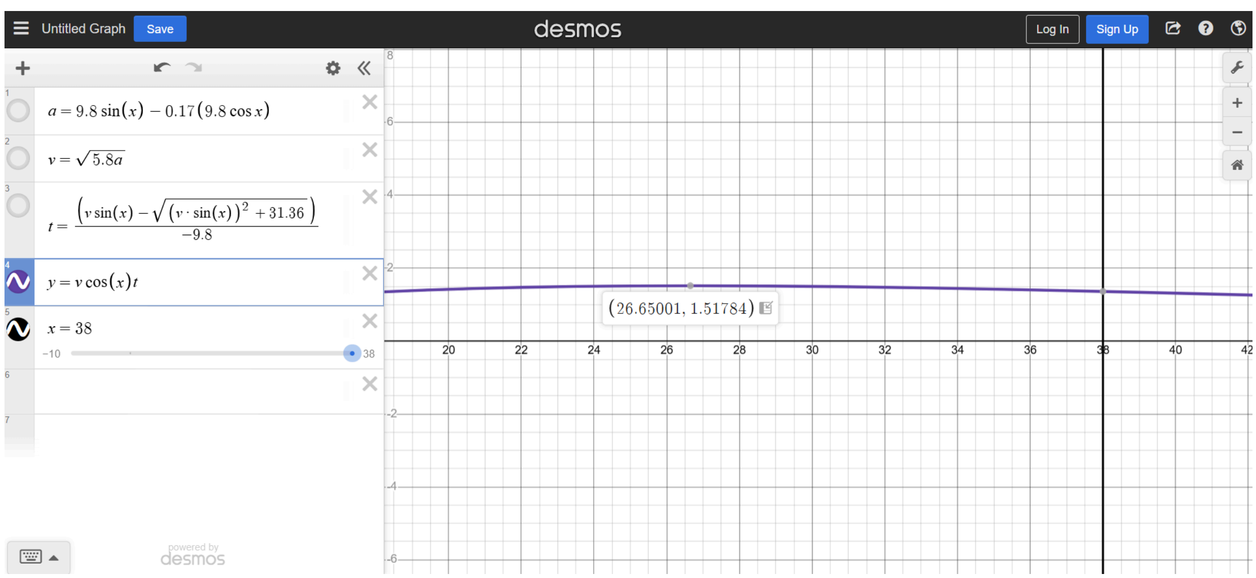
9. V is a vector and the V at the end of the inclined ramp would be equal to the initial velocity of the projectile after it flies off the ramp. Vo of projectile = 5.23m/s.
10. Now, we broke up the projectile problem into a horizontal and vertical component and solved for time first. A key part of this is that in the vertical direction, Vo = 5.23cosθ and

in the horizontal direction, V = 5.23sinθ. Then, we plugged in the time to solve for Δx. The work is shown below:



11. The puck lands 1.36 meters from the base of the counter.

Part B:



The maximum distance possible is 1.518 m with the initial slope at an angle of 26.65 degrees

Extensions

1. Suppose a 100 g weight is placed on top of the puck. How would this affect the angle for which the puck travels the farthest?

2. How far would the puck travel (in part a) if it were starting from an initial velocity of 10 m/s instead of from rest?
3. How far would the puck travel in part a if there was no friction?